Math 5040-1 Midterm
Fall 2008

Instructions:

- This exam is due Wednesday October 29, *in lecture*. Late exams are not accepted.

- You may not discuss this exam with other persons; this includes other students in this course. Failure to do this will result in a zero in this exam; further action might also be taken in accord with the university bylaws. By taking this exam you are agreeing that this represents your work alone.

Problems:

1. (Theoretical problem) Let $X := \{X_n\}_{n=0}^{\infty}$ denote a Markov chain on a finite state space S with transition matrix $\mathbf{P}$, and consider the stochastic process $Y := \{Y_n\}_{n=0}^{\infty}$ defined by setting $Y_k := X_{2k}$. Is Y a Markov chain? Prove or disprove carefully.

2. (Theoretical problem) Consider a Markov chain $X := \{X_n\}_{n=0}^{\infty}$ on the state space $S := \{1\,,2\,,3\}$ with the following transition matrix:

$$\mathbf{P} := \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

   And the initial probability distribution of X is $\bar{\phi}_0 := (1/3\,,1/3\,,1/3)$. For all integers $n \geq 0$ define the random variable Y_n to be the indicator of the event $\{X_n \neq 1\}$. That is, $Y_n := 0$ if $X_n = 1$ and $Y_n := 1$ otherwise. Is $Y := \{Y_n\}_{n=0}^{\infty}$ a Markov chain? Prove or disprove carefully.

3. (Simulation problem) A 52-card deck of card is thoroughly shuffled; then all of the cards are layed out in order [from left to right, say]. What is the probability that a King is set immediately next to an Ace?