p. 10, #7. Let \( p(i, j) \) denote the probability that we roll \( i \) dots on the first die and \( j \) dots on the second. Note that \( p(i, j) = \frac{1}{36} \) for every fixed choice of \( i, j = 1, \ldots, 6 \).

(a) We want
\[
p(1, 1) + p(1, 2) + p(2, 1) + p(2, 2) = \frac{4}{36} = \frac{1}{9}.
\]

(b) We want
\[
\text{The above expression} + p(1, 3) + p(2, 3) + p(3, 3) + p(3, 1) + p(3, 2)
= \frac{4}{36} + \frac{5}{36} = \frac{9}{36} = \frac{1}{4}.
\]

(c) We want
\[
p(1, 3) + p(2, 3) + p(3, 3) + p(3, 1) + p(3, 2) = \frac{5}{36}.
\]

(d) Let \( P(x) \) denote the probability that the maximum is exactly equal to \( x \). Then we work in steps:
\[
P(1) = p(1, 1) = \frac{1}{36};
\]
\[
P(2) = p(1, 2) + p(2, 1) + p(2, 2) = \frac{3}{36} \left[= \frac{1}{8}\right];
\]
\[
P(3) = \frac{5}{36}; \quad \text{[as before]}
\]
\[
P(4) = p(1, 4) + \cdots + p(4, 4) + p(4, 3) + \cdots + p(4, 1) = \frac{7}{36};
\]
\[ P(5) = p(1, 5) + \cdots + p(5, 5) + p(5, 4) + \cdots + p(5, 1) = \frac{9}{36} \left[ = \frac{1}{4} \right] ; \]

\[ P(6) = p(1, 6) + \cdots + p(6, 6) + p(6, 5) + \cdots + p(6, 1) = \frac{11}{36} . \]

We are asked to also compute \( Q(x) := \text{probability that the max is } x \text{ or less} \). Once again we work in steps:

\[ Q(1) = P(1) = \frac{1}{36} ; \]

\[ Q(2) = P(1) + P(2) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36} ; \]

\[ Q(3) = P(1)+P(2)+P(3) = Q(2)+P(3) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36} ; \]

\[ Q(4) = Q(3) + P(4) = \frac{9}{36} + \frac{7}{36} = \frac{16}{36} ; \]

\[ Q(5) = Q(4) + P(5) = \frac{16}{36} + \frac{9}{36} = \frac{25}{36} ; \]

\[ Q(6) = Q(5) + P(6) = \frac{25}{36} + \frac{11}{36} = 1 . \]

(e) 1, because the maximum of the two is one, and exactly one, of 1, \ldots, 6. Alternatively, \( Q(6) = 1 \). Yet another way to see this is

\[ P(1) + \cdots + P(6) = \frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{7}{36} + \frac{9}{36} + \frac{11}{36} = 1 . \]

p. 30–31, #2. (a) \((A \cap B^c) \cup (A^c \cap B)\); alternatively, \((A \setminus B) \cup (B \setminus A)\).

(b) \((A \cup B \cup C)^c\).

(c) i. Exactly one of \( A, B, \) or \( C \):

\[(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C).\]

ii. Exactly two of \( A, B, \) or \( C \):

\[(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C).\]

iii. Exactly three of \( A, B, \) or \([\text{and}]\) \( C \):

\[ A \cap B \cap C. \]
p. 30–31, #6.  i. That sentence has 10 words: One 1-letter word [“a”],
two 2-letter words [“is” and “at”], three 4-letter words
[“word,” “from,” and “this”], two 6-letter words [“picked”
and “random”], and two 7-letter words [“Suppose” and
“sentence”]. Therefore, the probabilities are distributed
as follows:

<table>
<thead>
<tr>
<th>Number of letters</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

ii. Similarly, six words have 1 vowel [“a,” “word,” “is,” “at,”
“from,” and “this”], two words have 2 vowels [“picked”
and “random”], and two words have 3 vowels [“Sup-
pose” and “sentence”] Therefore, the distribution of the
vowel counts is the following:

<table>
<thead>
<tr>
<th>Number of vowels</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

p. 45–46, #3.

\[
P(\text{rain tomorrow} | \text{rain today}) = \frac{P(\text{rain tomorrow and today})}{P(\text{rain today})} = \frac{3}{4}.
\]

p. 45–46, #4. Let us call the events \(A\) and \(B\), so that \(P(A) = 0.1\) and \(P(B) = 0.3\) [say].

(a) We are asked to compute \(1 - P(A \cup B)\). Since that \(P(A \cap B) = P(A) + P(B) - P(A \cap B)\), independent tells us that

\[
P(A \cap B) = 0.1 + 0.3 - (0.1 \times 0.3) = 0.37.
\]

Therefore, the answer is \(1 - 0.37 = 0.63\).

(b) \(P(A \cup B) = 0.37\); see the previous part.

(c) The answer is \(P(A \cap B^c) + P(A^c \cap B)\). By independence,

\[
P(A \cap B^c) = P(A)P(B^c) = 0.1 \times 0.7 = 0.07,
\]

and

\[
P(A^c \cap B) = P(A^c)P(B) = 0.9 \times 0.3 = 0.27.
\]

Therefore, the answer is \(0.07 + 0.27 = 0.34\).
This problem is worded loosely. There are many ways to interpret it. Here is one: You select a card at random with the given probabilities: respectively, 0.3 for double-white; 0.5 for black/white; and 0.2 for double-black cards. If you pick a card, then you put it face down on the black side, if it has one. We know that

\[ P(WW) = 0.3, \quad P(BW) = 0.5, \quad \text{and} \quad P(BB) = 0.2, \]

and want \( P(BW \mid B?) \) [the notation ought to be clear here]. We can apply the conditional-probability formula:

\[
P(BW \mid B?) = \frac{P(BW)}{P(B?)} = \frac{0.5}{P(BW) + P(BB)} = \frac{0.5}{0.5 + 0.2} = \frac{5}{7}.
\]

Now suppose we change the interpretation of the problem as follows: All three cards are sided [side 1 and side 2]. After you pick a card at random, you toss independently a coin. If that coin lands on heads then we choose side one; else, we select side 2. Then, as above,

\[
P(\text{black and white} \mid \text{black side showing}) = \frac{P(\text{black and white} \& \text{black side showing})}{P(\text{black side showing})}.
\]

But now the numerator is

\[
P(\text{black side showing} \mid \text{black and white}) \times P(\text{black and white}) = \frac{1}{4}.
\]

And the denominator is

\[
P(\text{black side showing} \mid \text{black and white}) \times P(\text{black and white}) + P(\text{black side showing} \mid \text{double black}) \times P(\text{double black})
\]
\[
+ P(\text{black side showing} \mid \text{double white}) \times P(\text{double white})
\]
\[
= \frac{1}{4} + 0.2 + 0 = 0.45.
\]

We apply the difference rule; viz.,

\[
P(F \mid G^c) = \frac{P(F \cap G^c)}{P(G^c)} = \frac{P(F) - P(FG)}{1 - P(G)}.
\]