

Solutions to Midterm #2
Mathematics 5010–1, Summer 2006
Department of Mathematics, University of Utah
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1.

(a) Let $p(x) := P\{X = x\}$. Then,

$$p(x) = \begin{cases} \frac{1}{10}, & \text{if } x = 1, 2, \dots, 10, \\ 0, & \text{otherwise.} \end{cases}$$

(b) $P\{X \geq 3\} = p(3) + \dots + p(10) = \boxed{8/10}$.

2. Define “success” to be “born in April or October” to see that X has the binomial distribution with parameters $n = 6$ and $p = 1/3$. Therefore,

$$P\{X = 3\} = \binom{6}{3} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 \approx \boxed{0.0536}.$$

3.

(a) $0, 1, \dots, 5$.

(b) Let $p(x) := P\{X = x\}$ to find that

$$p(x) = \begin{cases} \frac{\binom{5}{x}}{2^5}, & \text{if } x = 0, \dots, 5, \\ 0, & \text{otherwise.} \end{cases}$$

(c) $E(X) = \sum_{k=0}^5 k \binom{5}{k} 2^{-5} = \boxed{5/2}$. Also, $E(X^2) = \sum_{k=0}^5 k^2 \binom{5}{k} 2^{-5} = 30/4$, so that $\text{Var}(X) = (30/4) - (5/2)^2 = \boxed{5/4}$.

Extra Sheet for your work.