Solutions to Midterm #1
Mathematics 5010–1, Summer 2006
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1.

(a) $4! = 24$ permutations. Therefore, the probability is $\boxed{1/24}$.

(b) Let C_i denote the event that we guess correctly on trial number i. We know that $P(C_i) = 1/24$ from the first part, and wish to know

$$P(C_1^c \cap C_2^c \cap \cdots \cap C_n^c) = \boxed{\left(\frac{23}{24}\right)^n}.$$

2. Let H be the event that Julie is hemophilic; S_i the event that son number i is hemophilic. We know: $P(H) = 1/4$, and $P(S_1 \mid H) = P(S_2 \mid H) = 1/2$. We wish to know $P(S_1^c \cap S_1^c)$. By Bayes's formula,

$$\begin{aligned} P(S_1^c \cap S_2^c) &= P(S_1^c \cap S_2^c \mid H)P(H) + P(S_1^c \cap S_2^c \mid H^c)P(H^c) \\ &= P(S_1^c \mid H)P(S_2^c \mid H)P(H) + P(S_1^c \cap S_2^c \mid H^c)P(H^c) \\ &= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{4}\right) + \left(1 \times \frac{3}{4}\right), \end{aligned}$$

which is $\boxed{13/16}$.

3. $P(2 \mid \text{even}) = P(2 \text{ and even})/P(\text{even})$, which is

$$\frac{P(2)}{P(\text{even})} = \frac{1/6}{1/2} = \boxed{\frac{1}{3}}.$$

4. By Bayes' formula,

$$P(E) = P(A) + P(E \mid N)P(N) = \frac{4}{52} + P(E) \times \frac{36}{52}.$$

Therefore, $P(E) = \boxed{1/4}$.