

Math 5010–1, Summer 2006

Practice-Final Examination

1. Let $X_1, X_2, \dots, X_{20}$ be independent Poisson random variables with mean one.
 - (a) Compute $\text{Var}(X_1)$.
 - (b) Use the preceding and the central limit theorem to approximate $P\{\sum_{i=1}^{20} X_i > 15\}$. If you did not attempt part (a), you can answer this part by assuming that $\text{Var}(X_1) = 3$. Otherwise, you **must** use your answer to part (a) in order to merit partial credit.
2. A point (X, Y) is chosen at random according to the following (joint) probability mass function:

		Y		
		0	1	2
X	0	1/12	1/12	1/12
	1	1/12	1/12	1/12
	2	0	1/8	1/8
	3	0	0	1/4

- (a) Compute the mass function of $W = XY$.
 - (b) Compute $P\{X > Y\}$.
3. Suppose X and Y are independent, both distributed uniformly on $[0, 1]$. Then prove that

$$E(|X - Y|^\alpha) = \frac{2}{(\alpha + 1)(\alpha + 2)} \quad \text{for } \alpha > 0.$$

4. A fair die is cast ten times, and the total number of rolled dots is summed up; call this sum X . Find $E[X]$.
5. (a) Compute the moment generating function of a random variable X whose mass function is as follows:

$$p(0) = \frac{1}{3}, \quad p(2) = \frac{1}{6}, \quad p(-1) = \frac{1}{2}.$$

- (b) Suppose Y is a random variable whose moment generating function is

$$M_Y(t) = \frac{1}{2}e^t + \frac{1}{6}e^{-2t} + \frac{1}{3}e^{5t}.$$

Compute the mass function of Y .

6. You have 4 light bulbs whose lifetimes are independent normal random variables with mean 100 (hours) and standard deviation 5 (hours). Find the probability that your 4 light bulbs together live for at least 402 hours.

7. Let X denote a random variable with the following probability mass function.

$$p(j) = 2^{-j}, \quad j = 1, 2, \dots$$

- (a) Compute the moment generating function of X . (HINT: You may use the fact that $\sum_{j=0}^{\infty} r^j = 1/(1-r)$ for $0 < r < 1$.)
 - (b) Use your answer to part (a) to compute the expectation of X .
8. A point (X, Y) is picked at random, uniformly from the square whose corners are at $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(1, 1)$.
- (a) Compute $\text{Cov}(X, Y)$.
 - (b) Compute $P\{Y > \frac{3}{2} + X\}$.
9. A pair of fair dice are cast independently from one another. If you are told that the total sum of the dots is 10, find the probability that the first die was 6.