P314 #14

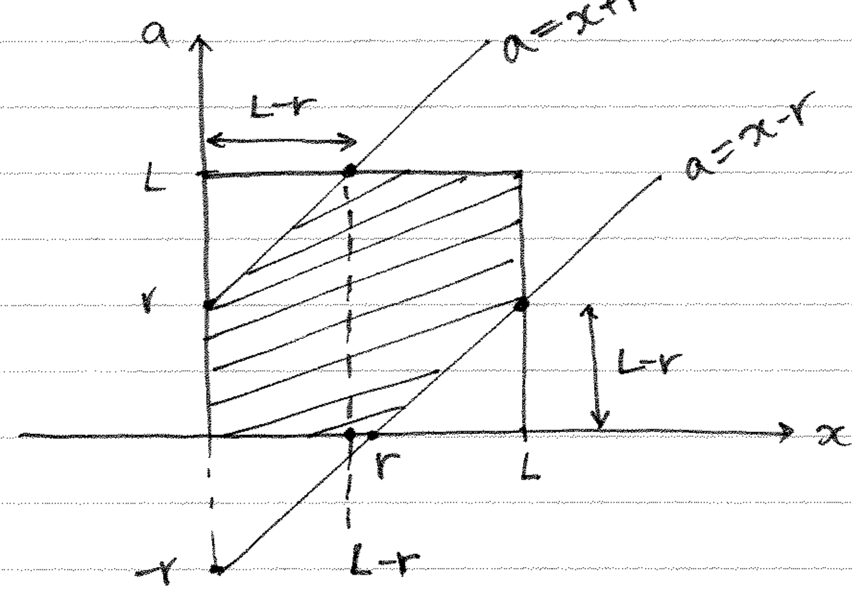
HW 7

[Addendum] The problem wanted the distribution of $|X-A|$ not just its expectation.

So 1st find cdf, then differentiate:

$$P\left\{|X-A| \le r\right\} = P\left\{-r \le A-X \le r\right\}$$

$$= P\left\{(X, A) \text{ falls in the shaded region}\right\}$$

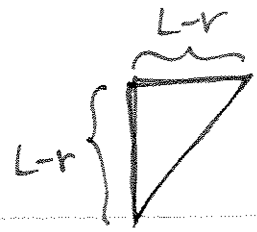


Now (X, A) is uniformly distributed on the square (independence)! Therefore,

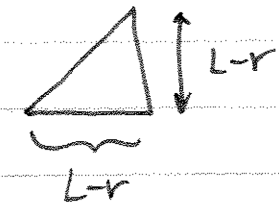
$$P\left\{|X-A| \le r\right\} = \frac{\text{area of shaded region}}{L^2}$$

$$= 1 - \frac{\text{area of unshaded region}}{L^2}$$

HW7 Addendum
p2

• area of 

is $\dfrac{(L-r)^2}{2}$

• area of 

is also $\dfrac{(L-r)^2}{2}$

$$\Rightarrow \quad P(|X-A| \le r) = 1 - \frac{(L-r)^2}{L^2} = 1 - \left(\frac{L-r}{L}\right)^2$$

$$= 1 - \left(1 - \frac{r}{L}\right)^2 \qquad 0 \le r \le L.$$

Differentiate [dr]:

$$f_{|X-A|}(r) = \begin{cases} 2\left(1 - \frac{r}{L}\right) \cdot \frac{1}{L} & 0 \le r \le L, \\ 0 & \text{o/w}. \end{cases}$$

N.B. $E(|X-A|) = \dfrac{2}{L} \displaystyle\int_0^L r\left(1 - \frac{r}{L}\right) dr$

$$= \frac{2}{L^2} \int_0^L r(L-r)\, dr$$

$$= \frac{2}{L^2} \left[L \int_0^L r\, dr - \int_0^L r^2\, dr\right]$$

$$= \frac{2}{L^2} \left[\frac{1}{2}L^3 - \frac{1}{3}L^3\right] = \frac{L}{3}, \text{ as before.}$$