Solutions to Homework #3
Mathematics 5010–1, Summer 2006
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Read this only after you have really thought hard about the problems.

Chapter 1 Problems

#27, p. 18. The answer is $\boxed{12!/(3!\cdot 4!\cdot 5!)}$. This is sometimes written as $\binom{12}{3,4,5}$.

#28, p. 18. Each teacher has 4 possible homes, so the total number of possible assignments = $\boxed{4^8}$. The second part is amusing though not assigned. The problem is: In how many ways can you divide 8 teachers into 4 groups of 2. Ans = $\binom{8}{2,2,2,2}$.

Chapter 2 Problems

#9, p. 56. Let $A = \{\text{American Express}\}$ and $V = \{\text{Visa}\}$. We know that $P(A) = 0.24$, $P(V) = 0.61$, and $P(A \cap V) = 0.11$. We want $P(A \cap V) = P(A) + P(V) - P(A \cap V) = 0.24 + 0.61 - 0.11 = \boxed{0.79}$.

#11, p. 56. In order to solve this you need to draw Venn diagrams.

Let $G = \{\text{cigarettes}\}$ and $C = \{\text{cigars}\}$. We know that $P(G) = 0.28$, $P(C) = 0.07$, and $P(C \cap G) = 0.05$.

(a) $P(C^c \cap G^c) = 1 - P(C \cap G) = 1 - \{P(C) + P(G) - P(C \cap G)\}$, which is $\boxed{0.7}$.

(b) $P(C \cap G^c) = P(C) - P(C \cap G) = \boxed{0.02}$.

#12, p. 56. Let S, F, and G designate the obvious events. You need to draw a Venn diagram to solve this problem. After you draw the Venn diagram you find that:

- $\#(S \cap F \cap G) = 2$;
- $\#(S \cap G \cap F^c) = 2$;
- $\#(S \cap F \cap G^c) = 10$;
- $\#(S \cap F^c \cap G^c) = 14$;
- $\#(S^c \cap F \cap G) = 4$;
- $\#(S^c \cap F \cap G^c) = 10$;
- $\#(S^c \cap F^c \cap G) = 8$.

The total is $2 + 2 + 10 + 14 + 4 + 10 + 8 = 50$. This is how many people take language classes.

(a) Pr = $50/100 = \boxed{0.5}$.

(b) Pr = $(14+10+8)/100 = \boxed{0.32}$.

(c) Pr = $1 - P\{\text{neither is taking languages}\} = 1-(50\times 49)/(100\times 99) = \boxed{0.75\overline{25}}$.

Chapter 3 Problems

#8, p. 112. Let G_i denote the event that the ith child is a girl. Assume that every possible two-child combination is equally likely. Then, $P(G_1 \cap G_2 \,|\, G_2) = P(G_2 \cap G_1)/P(G_1) = (1/4)/P(G_1)$. But

$$P(G_1) = P(G_1 \cap G_2) + P(G_1 \cap G_2^c) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

Therefore, $P(G_1 \cap G_2 \,|\, G_2) = (1/4)/(1/2) = \boxed{0.5}$.

#12, p. 112. Let $F = \{\text{passes the first exam}\}$, etc. We know that $P(F) = 0.9$, $P(S\,|\,F) = 0.8$, and $P(T\,|\,F\cap S) = 0.7$.

(a) $P(F\cap S\cap T) = P(T\,|\,F\cap S)P(F\cap S) = P(T\,|\,F\cap S)P(S\,|\,F)P(F) = 0.9\times 0.8\times 0.7 = \boxed{0.504}$.

(b) We want $P(S^c\,|\,(F\cap S\cap T)^c)$. Note that $S^c \cap (F\cap S\cap T)^c = F\cap S^c$. Therefore,

$$P\left(S^c \,\middle|\, (F\cap S\cap T)^c\right) = \frac{P(F\cap S^c)}{1-P(F\cap S\cap T)} = \frac{P(F\cap S^c)}{1-0.504}$$

But $P(F\cap S^c) = P(S^c\,|\,F)P(F) = 0.2\times 0.9 = 0.18$. Therefore,

$$P\left(S^c \,\middle|\, (F\cap S\cap T)^c\right) = \frac{0.18}{0.496} \approx \boxed{0.3629}.$$