Solutions to Homework #2
Mathematics 5010–1, Summer 2006
Department of Mathematics, University of Utah
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Problem 17, Chapter 1 from Assignment 1: The solution to this one was missing, so it is added here.
Experiment #1: You have to choose 7 children to distribute the gifts to—$\binom{10}{7}$ ways to do this.
Experiment #2: Distribute 7 gifts, one to each of the chosen children—7! ways.
Total=$7!\binom{10}{7} = \boxed{10!/3!}$.

Chapter 2 Problems

#15, p. 57. (a) Choose the suit (4 ways) and then distribute the cards ($\binom{13}{5}$ ways). Pr = $\boxed{4\binom{13}{5}/\binom{52}{5}}$.

(b) Choose the pair (13 ways) and distribute them ($\binom{4}{2}$ ways). Then choose the other 3 ($\binom{12}{3}$ ways), and distribute them (4^3 ways). Pr = $\boxed{13\binom{4}{2}\binom{12}{3}4^3/\binom{52}{5}}$.

(c) Choose the pair faces ($\binom{13}{2}$ ways) and distribute them ($\binom{4}{2}^2$ ways). Then choose and distribute the last card (44 ways). Pr = $\boxed{\binom{13}{2}\binom{4}{2}^2 44/\binom{52}{5}}$.

(d) Choose the triple (13 ways) and distribute them ($\binom{4}{3}$ ways). Then choose the other two ($\binom{12}{2}$) and distribute them (4^2 ways). Pr = $\boxed{13\binom{4}{3}\binom{12}{2}4^2/\binom{52}{5}}$.

(e) Choose the four (13 ways) and distribute them (1 way). Then choose and deal the other (48 ways). Pr = $\boxed{13\times 48/\binom{52}{5}}$.

Chapter 2 Theoretical Problems

#15, p. 62. Choose k white balls ($\binom{M}{k}$) and r black balls ($\binom{N}{r}$). Pr = $\boxed{\binom{M}{k}\binom{N}{r}/\binom{M+N}{k+r}}$.

Chapter 3 Problems

#7, p. 112. Let G_i denote the event that the ith child is a girl. Note that the event $\{$at least one boy$\}$ is the complement of $G_1 \cap G_2$. Therefore,

$$P\{\text{at least one boy}\} = 1 - P(G_1 \cap G_2) = 1 - \frac{1}{4} = \frac{3}{4}.$$

On the other hand,

$$P\{\text{exactly one boy}\} = P(G_1 \cap G_2^c) + P(G_1^c \cap G_2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

Hence,

$$P\left(\text{exactly one boy} \,\middle|\, \text{at least one boy}\right) = \frac{P\left(\{\text{exactly one boy}\} \cap \{\text{at least one boy}\}\right)}{P\{\text{at least one boy}\}}$$
$$= \frac{P\{\text{exactly one boy}\}}{P\{\text{at least one boy}\}} = \frac{1/2}{3/4} = \boxed{\frac{2}{3}}.$$

[Why is this answering the question?]

#15, p. 112. Let $S = \{\text{smoker}\}$ and $E = \{\text{ectopic pregnancy develops}\}$. We know that $P(E \mid S) = 2P(E \mid S^c)$. We also know that $P(S) = 0.32$. We wish to know $P(S \mid E)$.

We observe that $P(E \mid S) = P(E \cap S)/P(S) = P(E \cap S)/0.32$. Also, $P(E \mid S^c) = P(E \cap S^c)/0.68$. Therefore,

$$\frac{P(E \cap S)}{0.32} = 2 \times \frac{P(E \cap S^c)}{0.68} = \frac{P(E \cap S^c)}{0.34}.$$

On the other hand, $P(E \cap S) + P(E \cap S^c) = P(E)$. Therefore,

$$\frac{P(E \cap S)}{0.32} = \frac{P(E)}{0.34} - \frac{P(E \cap S)}{0.34}.$$

Equivalently,

$$P(E \cap S) \times \frac{0.66}{0.32 \times 0.34} = P(E \cap S)\left[\frac{1}{0.32} + \frac{1}{0.34}\right] = \frac{P(E)}{0.34}.$$

Equivalently,

$$P(S \cap E) = P(E) \times \frac{0.32}{0.66}.$$

Hence,

$$P(S \mid E) = \frac{0.32}{0.66} = \boxed{0.\overline{48}}.$$

#20, p. 113. Let $F = \{\text{female}\}$ and $C = \{\text{computer science majors}\}$. We know that $P(F) = 0.52$, $P(C) = 0.05$, and $P(F \cap C) = 0.02$.

(a) $P(F \mid C) = P(F \cap C)/P(C) = 0.02/0.05 = \boxed{2/5}$.

(b) $P(C \mid F) = P(F \cap C)/P(F) = 0.02/0.52 = \boxed{1/16}$.

#26, p. 114. Let $M = \{\text{man}\}$ and $C = \{\text{colorblind}\}$. We know that $P(M) = 0.5$, $P(C \mid M) = 0.05$, and $P(C \mid M^c) = 0.0025$. Then

$$P(M \mid C) = P(C \mid M)\frac{P(M)}{P(C)} = 0.05 \times \frac{0.5}{P(C \mid M)P(M) + P(C \mid M^c)P(M^c)}$$
$$= 0.05 \times \frac{0.5}{0.05 \times 0.5 + 0.0025 \times 0.5} \approx \boxed{0.95238}.$$

#38, p. 115. Let T and W stand for the obvious events. We know that $P(T) = 0.5$, $P(W \mid T^c) = 5/12$, and $P(W \mid T) = 3/15 = 1/5$. Then,

$$P(T \mid W) = P(W \mid T)\frac{P(T)}{P(W)} = \frac{1}{5} \times \frac{0.5}{P(W)} = \frac{1}{10P(W)}.$$

By Bayes' formula,

$$P(W) = P(W \mid T)P(T) + P(W \mid T^c)P(T^c) = \left(\frac{1}{5} \times 0.5\right) + \left(\frac{5}{12} \times 0.5\right) = \frac{9}{40}.$$

Therefore, $P(T \mid W) = \boxed{4/9}$.

#59, p. 118. (a) $P(\text{HHHH}) = \boxed{p^4}$.

(b) $P(\text{THHT}) = \boxed{p^2(1-p)^2}$.

(c) The only way that HHHH occurs before THHH is if we roll HHHH in the first tosses. So the answer is the $\boxed{\text{same as (a)}}$.