Solutions to Homework #1
Mathematics 5010–1, Summer 2006
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Problems

p. 16, #1. **(a)** There are 26 choices for the first two spots; there are 10 for the other 5. Thus, there are a total of $\boxed{26^2 \times 10^5} = 67,600,000$-many such license plates possible.

(b) $\boxed{26 \times 25 \times 10 \times 9 \times 8 \times 7 \times 6} = 19,656,000$-many.

p. 16, #3. "Lay out the jobs" in a row, and send the workers to them. There are $\boxed{20!} \approx 1.4 \times 10^{18}$-many different ways of doing this.

p. 16, #7. **(a)** $\boxed{6!} = 720$; the sexes are redundant information.

(b) We must either have BGBGBGBG or GBGBGBGB. Each has $3! \times 3! = 36$-many different possible outcomes. Therefore, there are $\boxed{72}$ possible outcomes altogether.

(c) We must have either BBBGGG, GBBBGG, GGBBBG, or GGGBBB. Each has $3! \times 3! =$ 36-many different possible ways. Therefore, there are a total of $4 \times 36 = \boxed{144}$-many different such boy-girl arrangements.

(d) This is the same question as (b); it also has the same answer.

p. 17, #13. If you were to write out all possible handshakes you could choose two people and make them shake hands, and then record it. Thus, the number of distinct possible handshakes is $\boxed{\binom{20}{2}} = 20 \times 19/2 = 190$.

p. 17, #21. Any path from A to B needs precisely 7 steps; in each step you use the direction $\rightarrow$ or $\uparrow$; you need a total of four $\rightarrow$'s and three $\uparrow$'s. Thus, the number of such paths is the same as the number of places—among seven—to put the $\rightarrow$'s [or $\uparrow$'s for that matter], and that is $\binom{7}{4} = \binom{7}{3}$, which is $\boxed{35}$.

Theoretical Exercises

p. 19, #8. In this exercise, n and m are assumed to be positive integers, and r is a positive integer that is less than or equal to m and n. Note that:

0. There are $\binom{n}{0}\binom{m}{r}$-many teams that have exactly zero men in them.

1. There are $\binom{n}{1}\binom{m}{r-1}$-many teams that have exactly one man in them.

2. There are $\binom{n}{2}\binom{m}{r-2}$-many teams that have exactly two men in them.

$\vdots$

$\boldsymbol{r}$**.** There are $\binom{n}{r}\binom{m}{0}$-many teams that have exactly r men in them.

Therefore the total number of different teams of r people is

$$\binom{n}{0}\binom{m}{r} + \cdots + \binom{n}{r}\binom{m}{0}.$$

On the other hand, we know that the number of teams of size r is $\binom{n+m}{r}$. Therefore, the display must be equal to $\binom{n+m}{r}$, whence follows the assertion of the exercise.