

Sambutan rami

$$P_{j+1} - P_{-M+1} = \frac{q}{p} P_{-M+1} \sum_{k=0}^{M+j-1} \left(\frac{q}{p}\right)^k$$

$$\sum_0^N r^k = \frac{1-r^{N+1}}{1-r} \quad \& \circ$$

$$P_{j+1} - P_{-M+1} = \frac{q}{p} P_{-M+1} \cdot \frac{1 - (q/p)^{M+j}}{1 - (q/p)}$$

$$P_{j+1} = P_{-M+1} \left[1 + \frac{q}{p} \frac{1 - (q/p)^{M+j}}{1 - (q/p)} \right]$$

$$= P_{-M+1} \cdot \left[1 + \frac{q - q(q/p)^{M+j}}{p - q} \right]$$

$$= P_{-M+1} \cdot \frac{p - q(q/p)^{M+j}}{p - q} \quad \oplus$$

$$1 = P_N = P_{-M+1} \cdot \frac{p - q(q/p)^{M+N-1}}{p - q}$$

$$\Rightarrow P_{-M+1} = \frac{p - q}{p - q(q/p)^{M+N-1}}$$

$$\text{By } \oplus \quad P_0 = \frac{p - q}{p - q(q/p)^{M+N-1}} \times \frac{p - q(q/p)^{M-1}}{p - q}$$

$$= \frac{p - q(q/p)^{M-1}}{p - q(q/p)^{M+N-1}}$$

$$= \frac{p^M - q^M}{p^M - q^{M+N} / p^N} = \frac{p^M - q^M}{p^M - q^M \left(\frac{q}{p}\right)^N}$$

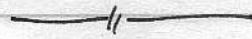
If $p > \frac{1}{2}$, then $q/p < 1$ and $(q/p)^N \rightarrow 0$ as $N \rightarrow \infty$.

$$\text{So } P_0 \rightarrow \frac{p^M - q^M}{p^M} = 1 - \left(\frac{q}{p}\right)^M$$

= prob. of winning ∞ before losing M .

Eg. $P\{\text{ever losing}\} = 1 - \left(1 - \frac{q}{p}\right) = q/p$.

If $p < \frac{1}{2}$, by symmetry, then $P(\text{ever winning}) = p/q$.



Ex- [Laplace Rule of Succession]

$k+1$ coins in a box. The i th flips H with prob i/k . $0 \leq i \leq k$.

Choose a coin at random and flip it n times independently.

If the n are all H 's, what is the coin's prob. that the next is H ?

$$C_i = \{\text{coin } i\} \quad F_i = \{\text{first } i \text{ are } H\}$$

$$P(H|F_n) = P(F_{n+1}|F_n) = \sum_{j=0}^k \frac{j}{k} P(F_{n+1}|F_n C_j) P(C_j|F_n)$$

$$P(F_{n+1}|F_n) = \sum_{j=0}^k P(F_{n+1}|F_n C_j) P(C_j|F_n) \quad [\text{why?}]$$

$$= \sum_{j=0}^k P(H|C_j) P(C_j|F_n)$$

$$= \sum_{j=0}^k \left(\frac{j}{k}\right) P(C_j|F_n) \quad \text{cancel out } P(F_n)$$

$$P(C_j|F_n) = \frac{P(F_n|C_j) P(C_j)}{P(F_n)} = \frac{(j/k)^n \frac{1}{k+1}}{P(F_n)}$$

$$P(F_n) = \sum_{j=0}^k P(F_n | G_j) P(G_j) = \frac{1}{k+1} \sum_{j=0}^k \left(\frac{j}{k}\right)^n.$$

$$\therefore P(G_j | F_n) = \frac{\left(\frac{j}{k}\right)^n \frac{1}{k+1}}{\frac{1}{k+1} \sum_{j=0}^k \left(\frac{j}{k}\right)^n} = \frac{\left(\frac{j}{k}\right)^n}{\sum_{j=0}^k \left(\frac{j}{k}\right)^n}$$

$$\therefore P(F_{n+1} | F_n) = \frac{\sum_{j=0}^k \left(\frac{j}{k}\right)^{n+1}}{\sum_{j=0}^k \left(\frac{j}{k}\right)^n}$$

$$\xrightarrow{k \rightarrow \infty} \frac{\int_0^1 x^{n+1} dx}{\int_0^1 x^n dx} = \frac{n+1}{n+2}.$$

Example (The problem of the points)

Bernoulli trials $p = P(\text{Success})$.

$P(n \text{ Successes before } m \text{ failures}) = ?$

Look at the first $n+m-1$ trials. n Success before m failure iff \exists n or more successes in the 1st $n+m-1$ trials.

$$\begin{aligned} \therefore \text{prob. } n \text{ successes before } m \text{ failures} &= P(n) + P(n+1) + \dots + P(n+m-1) \\ &= \binom{n+m-1}{n} p^n (1-p)^{m-1} + \dots + \binom{n+m-1}{n+m-1} p^{n+m-1} (1-p)^0 \\ &= \sum_{j=n}^{n+m-1} \binom{n+m-1}{j} p^j (1-p)^{n+m-1-j}. \end{aligned}$$