Read Chapter 7, sections 7.1–7.4, and review conditional expectation by reading through section 7.5 (edition 7).

The following are mainly borrowed from your text.

**Problems:**

1. A player throws a fair die and simultaneously flips a fair coin. If the coin lands heads then she wins twice, and if tails, then one-half of the value that appears on the die. Determine her expected winnings.

2. If $X$ and $Y$ are independent uniform-(0, 1) random variables, then prove that

   $$E(|X - Y|^\alpha) = \frac{2}{(\alpha + 1)(\alpha + 2)}$$

   for all $\alpha > 0$.

3. A group of $n$ men and $n$ women are lined up at random.
   (a) Find the expected number of men who have a woman next to them.
   (b) Repeat part (a), but now assume that the group is randomly seated at a round table.

4. Let $X_1, X_2, \ldots$ be independent with common mean $\mu$ and common variance $\sigma^2$. Set

   $$Y_n = X_n + X_{n+1} + X_{n+2}$$

   for all $n \geq 1$.

   Compute $\text{Cov}(Y_n, Y_{n+j})$ for all $n \geq 1$ and $j \geq 0$.

**Theoretical Problems:**

1. Suppose $X$ is a nonnegative random variable with density function $f$. Prove that

   $$E(X) = \int_0^\infty P\{X > t\} dt.$$  \hspace{1cm} (eq.1)

   Is this still true when $P\{X < 0\} > 0$? If “yes,” then prove it. If “no,” then construct an example.

2. (Hard) Suppose $X_1, \ldots, X_n$ are independent, and have the same distribution. Then, compute $\phi(x)$ for all $x$, where

   $$\phi(x) := E [X_1 | X_1 + \cdots + X_n = x].$$