Reading and Problem Assignment #8
Math 501–1, Spring 2006
University of Utah

Read sections 6.1-6.5 of Chapter 6 (edition 7). This is where the midterm material ends. So it may pay to get started early.

The following are borrowed from your text.

Problems:

1. A man and a woman agree to meet at a certain location at about 12:30 p.m. If the man arrives at a time uniformly distributed between 12:15 and 12:45, and if the woman independently arrives at a time uniformly distributed between 12:00 and 1 p.m., then find the probability that the first to arrive waits longer than 7 minutes. What is the probability that the man arrives first?

2. When a current I (in amperes) flows through a resistance R (in ohms), the power generated is given by $W = I^2R$ (in watts). Suppose that I and R are independent random variables with densities

$$\begin{aligned} f_I(x) &= 6x(1-x) \qquad && 0 \le x \le 1, \\ f_R(x) &= 2x && 0 \le x \le 1. \end{aligned}$$

Then find the density of W. Use this to compute EW.

3. Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the ith ball selected is white, and let X_i equal 0 otherwise. Compute the conditional probability mass function of X_1 given that $X_2 = 1$. Use this to find $E[X_1 \mid X_2 = 1]$.

4. The density of $(X\,, Y)$ is

$$f(x\,, y) = \begin{cases} xe^{-x(y+1)}, & \text{if } x > 0 \text{ and } y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the conditional density of X, given $Y = y$. Use this to compute $E[X \mid Y = y]$.
(b) Find the conditional density of Y, given $X = x$. Use this conditional density to compute $E[Y \mid X = x]$.
(c) Find the density of $Z = XY$. Use this to compute $E[Z]$ and $\mathrm{Var}(Z)$.

Theoretical Problems:

1. Suppose X is exponentially distributed with paramater $\lambda > 0$. Find

$$P\left\{[X] = n\ ,\ X - [X] \le x\right\},$$

where $[a]$ denotes the largest integer $\leq a$. Are X and $X - [X]$ independent?

2. Suppose X and Y are independent, standard normal random variables. Prove that $Z := X/Y$ has the Cauchy density. That is, prove that the density of Z is

$$f_Z(a) = \frac{1}{\pi(1+a^2)}, \qquad -\infty < a < \infty.$$

(`Hint:` Consider first $P\{X \leq tY\}$.)

3. Suppose X is a standard normal random variable. Compute the density of $Y = X^2$. [The resulting density is called the chi-squared distribution with one degree of freedom. It is sometimes also written as χ_1^2. Warning: "chi" is read as "kye," and not "chai."]