

Reading and Problem Assignment #7
Math 501–1, Spring 2006
University of Utah

Read the section on “The distribution of a function of a random variable” in Chapter 5. (Section 5.7 in edition 7). Also start reading sections 6.1-6.2 of Chapter 6 (jointly distributed random variables; edition 7).

The following are borrowed from your text.

Problems:

1. Suppose Y is uniformly distributed on $(0, 5)$. What is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real?
2. Two fair dice are rolled. Find the joint mass function of (X, Y) when:
 - (a) X is the maximum (i.e., largest) of the values of the two dice, and Y is the sum of the values of the two dice;
 - (b) X is the value of the first die and Y is the maximum of the values of the two dice;
 - (c) X is the minimum (i.e., smallest) of the values of the two dice, and Y is the maximum of the two values.
3. Consider a sequence of independent Bernoulli trials, each of which is a success with probability p . Let X_1 denote the number of failures preceding the first success, and let X_2 be the number of failures between the first two successes. Find the joint mass function of (X_1, X_2) .
4. The joint density function of (X, Y) is given by

$$f(x, y) = \begin{cases} c(y^2 - x^2)e^{-y}, & \text{if } -y \leq x \leq y \text{ and } 0 < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find c .
 - (b) Find the (marginal) density functions of X and Y respectively.
 - (c) Find $E(X)$.
 - (d) Find $P\{X > Y\}$.
5. The (joint) density function of (X, Y) is given by

$$f(x, y) = \begin{cases} e^{-(x+y)}, & \text{if } 0 \leq x < \infty, \text{ and } 0 \leq y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

Find: (a) $P\{X < Y\}$; and (b) $P\{X < a\}$ for all real numbers a .