Reading and Problem Assignment #7  
Math 501–1, Spring 2006  
University of Utah

Read the section on “The distribution of a function of a random variable” in Chapter 5. (Section 5.7 in edition 7). Also start reading sections 6.1-6.2 of Chapter 6 (jointly distributed random variables; edition 7).

The following are borrowed from your text.

Problems:

1. Suppose $Y$ is uniformly distributed on $(0, 5)$. What is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real?

2. Two fair dice are rolled. Find the joint mass function of $(X, Y)$ when:
   (a) $X$ is the maximum (i.e., largest) of the values of the two dice, and $Y$ is the sum of the values of the two dice;
   (b) $X$ is the value of the first die and $Y$ is the maximum of the values of the two dice;
   (c) $X$ is the minimum (i.e., smallest) of the values of the two dice, and $Y$ is the maximum of the two values.

3. Consider a sequence of independent Bernoulli trials, each of which is a success with probability $p$. Let $X_1$ denote the number of failures preceding the first success, and let $X_2$ be the number of failures between the first two successes. Find the joint mass function of $(X_1, X_2)$.

4. The joint density function of $(X, Y)$ is given by

   \[ f(x, y) = \begin{cases} 
   c(y^2 - x^2)e^{-y}, & \text{if } -y \leq x \leq y \text{ and } 0 < y < \infty, \\
   0, & \text{otherwise.} 
   \end{cases} \]

   (a) Find $c$.
   (b) Find the (marginal) density functions of $X$ and $Y$ respectively.
   (c) Find $E(X)$.
   (d) Find $P\{X > Y\}$.

5. The (joint) density function of $(X, Y)$ is given by

   \[ f(x, y) = \begin{cases} 
   e^{-(x+y)}, & \text{if } 0 \leq x < \infty, \text{ and } 0 \leq y < \infty, \\
   0, & \text{otherwise.} 
   \end{cases} \]

   Find: (a) $P\{X < Y\}$; and (b) $P\{X < a\}$ for all real numbers $a$. 