Reading and Problem Assignment #5
Math 501–1, Spring 2006
University of Utah

Read Chapter 4 (expectations). Start reading Chapter 5 (continuous random variables).

The following are borrowed from your text.

Problems:

1. A gambling book recommends the following "winnin strategy" for the game of roulette: It recommends that the gambler bet \$1 on red. If red appears (this has probability 18/38), then the gambler should take her \$1 profit and quit. Else, she should make additional \$1 bets on red on each of the next two spins of the roulette wheel, and then quit. Let X denote the gambler's winnings when she quits.
(a) Find $P\{X > 0\}$.
(b) Are you convinced that the strategy is indeed a "winning" strategy? Explain your answer.
(c) Compute EX.

2. One of the numbers 1 through 10 is chosen at random. You are to try and guess the number chosen by asking questions with "yes-no" answers. Compute the expected number of questions that you need to ask in each of the following two cases:
(a) Your ith question is to be, "Is it i?", for $i = 1, \ldots, 10$.
(b) With each question you try to eliminate one-half of the remaining numbers, as nearly as possible. [E.g., "Is it greater than or equal to 5?", etc.]

3. A person tosses a fair coin until a tail appears for the first time. If "tail" appears on the nth flip, then the person wins 2^n dollars. Let X denote the person's winnings. Show that $EX = \infty$. This is known as the "*St.-Petersbourg paradox.*"

4. A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you win −\$1.00 (i.e., you lose one dollar). Compute:
(a) the expected value of the amount you win;
(b) the variance of the amount you win.

Theoretical Problems:

1. Let X be such that $P\{X = 1\} = p$ and $P\{X = -1\} = 1 - p$. Find a constant $c \neq 1$ such that $E[c^X] = 1$.

2. Prove that if X is a Poisson random variable with parameter λ, then for all $n \geq 1$,

$$E[X^n] = \lambda E[(X+1)^{n-1}].$$

Use this to compute EX, $E[X^2]$, $\mathrm{Var}X$, and $E[X^3]$.