Reading and Problem Assignment #4
Math 501–1, Spring 2006
University of Utah

Read Chapter 4 (random variables).

The following are borrowed from your text.

Problems:

1. Suppose the distribution function of X is given by

$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1/2, & \text{if } 0 \le x < 1, \\ 3/5, & \text{if } 1 \le x < 2, \\ 4/5, & \text{if } 2 \le x < 3, \\ 9/10, & \text{if } 3 \le x < 3.5, \\ 1, & \text{if } x \ge 3.5. \end{cases}$$

Compute the mass function of X.

2. Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares with player 3; and so on. Let X denote the number of times player 1 is declared a winner. Find the mass function of X. Use this to find the probability that X is even.

3. Four independent flips of a fair coin are made. Let X denote the number of heads so obtained. Find the mass function of $X - 2$, and plot this function.

4. A fair coin is continually flipped until heads appear for the tenth time. Let X denote the number of tails that occur in the mean time. Compute $P\{2 \le X \le 5\}$.

Theoretical Problems:

1. Let X be a negative binomial random variable with parameters r and p, and let Y be a binomial random variable with parameters n and p. Prove that $P\{X > n\} = P\{Y < r\}$.

2. Suppose X is geometric with parameter p. That is, $P\{X = k\} = p(1-p)^{k-1}$ for $k = 1, 2, \ldots$. Then:
(a) Compute explicitly $P\{X \ge n\}$ for any positive integer n.
(b) Compute explicitly $P\{X = n + k \,|\, X > n\}$. Compare your answer to $P\{X = k\}$.