Reading and Problem Assignment #1
Math 501–1, Spring 2006
University of Utah

Read chapter 1 (combinatorial analysis). Skip the starred sections.

The following are borrowed from your text.

Problems:

1. Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible?

2. Consider a group of 20 people. If everyone shakes hands with everyone else, then how many handshakes take place?

3. Five separate awards (best scholarship, best leadership qualities, and so on ) are to be presented to selected students from a class of 30. How many different outcomes are possible if:
   (a) a student can receive any number of awards;
   (b) each student can receive at most 1 award?

4. A person has 8 friends, of whom 5 will be invited to a party.
   (a) How many choices are there if 2 of the friends are feuding and will not attend together?
   (b) How many choices if 2 of the friends will only attend together?

Theoretical Problems:

1. Verify that \( \binom{n}{k} = \binom{n}{n-k} \). Use this to prove that

\[
\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.
\]

[Hint for the second part: In order to choose \( n \) from \( 2n \) people, you must choose some number \( k \in \{0, \cdots, n\} \) from the first \( n \) people.]