Solutions to Problem Assignment #2
Math 501–1, Spring 2006
University of Utah

Problems:

1. *Consider n-digit numbers where each digit is one of the 10 integers* $0, 1, \ldots, 9$. *How many such numbers are there for which:*
(a) *No two consecutive digits are equal?*
(b) 0 *appears as a digit a total of* i *times,* $i = 0, \ldots, n$*?*

Solution: For **(a)** we have $10 \times 9^{n-1}$. For **(b)**, choose the i spots first, and then dispurse 1-9 in the remaining $n - i$ spots. Answer: $\binom{n}{i} \times 9^{n-i}$.

2. *A student is to answer 7 out of 10 questions in an examination. How many choices has she? How many if she must answer at least 3 of the first 5 questions?*

Solution: She has $\binom{10}{7}$ choices. In the second case, the answer is $\binom{5}{3}\binom{5}{4} + \binom{5}{4}\binom{5}{3} + \binom{5}{5}\binom{5}{2}$.

3. *If 8 new teachers are to divided among 4 schools, then how many divisions are possible? What if each school must receive at least 2 new teachers?*

Solution: For the first part, distribute the schools to the teachers: Number of distinct ways is 4^8. For the second part, you assign 2 teachers per school. Number of distinct ways is $\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}$.

4. *If all* $\binom{52}{5}$ *possible poker hands are equally likely, then what is the probability of:*

(a) *a flush? [This is when all cards have the same suit; e.g.,* $A\clubsuit, 2\clubsuit, 5\clubsuit, J\clubsuit, K\clubsuit$*.]*

Solution: First choose the suit (4 ways), and then the cards ($\binom{13}{5}$ ways). Answer: $4\binom{13}{5}$.

(b) *one pair? [This is when the cards have the denominations* a, a, b, c, d*, where* a*,* b*,* c*, and* d *are all distinct. E.g.,* $A\spadesuit, A\clubsuit, 2\spadesuit, 3\clubsuit, Q\heartsuit$*.]*

Solution: Choose the pair (13 ways), deal it ($\binom{4}{2}$ ways), choose the other three ($\binom{12}{3}$), and deal them (4^3). This yields $13\binom{4}{2}\binom{12}{3}4^3$.

(c) *four of a kind? [This is when the cards have denominations* a, a, a, a, b*. E.g.,* $A\spadesuit, A\clubsuit, A\heartsuit, A\diamondsuit, 10\heartsuit$*.]*

Solution: Choose and deal the "four" (13 ways), and then choose and deal the other one (48 ways). This yields $13 \times 48 = 624$.