Math 4200, Summer '09, HW sol #1

#4 (p.8)

$$\frac{2-3i}{3+2i} = \frac{2-3i}{3+2i} \cdot \frac{3-2i}{3-2i}$$

$$= \frac{(2-3i)(3-2i)}{|3+2i|^2}$$

$$= \frac{-13i}{9+4} = -i.$$

#5 (p.8)

$$i^{1/2} = |i|^{1/2} e^{i\frac{\arg i}{2} + i\frac{2\pi k}{2}} \quad \text{for } k=0,1$$

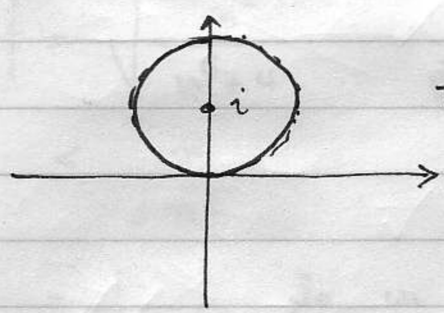
$\arg i = \frac{\pi}{2}$, so

$$\sqrt{i} = e^{i\pi/4 + i\pi k} \quad \text{for } k=0,1$$

roots are $= e^{i\pi/4}$ and $e^{i5\pi/4}$

$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ and $\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)$.

#13 (p.8)



Disc of radius 1 about i.

#10 (p.15) $\sum_{n=1}^{\infty} \frac{n}{3+2ni}$ converges if and only if

$\sum_{n=1}^{\infty} \operatorname{Re}\left(\frac{n}{3+2ni}\right)$ and $\sum_{n=1}^{\infty} \operatorname{Im}\left(\frac{n}{3+2ni}\right)$ both converge.

Note that the conjugate to $\frac{n}{3+2ni}$ is $\frac{n}{3-2ni}$. Therefore,

$$\operatorname{Re}\left(\frac{n}{3+2ni}\right) = \frac{1}{2}\left\{\frac{n}{3+2ni} + \frac{n}{3-2ni}\right\}$$

$$= \frac{1}{2}\,\frac{6n}{(3+2ni)(3-2ni)} = \frac{3n}{9+4n^2}.$$

As $n \to \infty$, this behaves like $\frac{3}{4n}$. Therefore $\sum \operatorname{Re}(14)$ is <u>not</u> convergent.

<u>#11 (p.15)</u> We'll show absolute convergence:

$$\left|\frac{n}{n^3+2i}\right|^2 = \frac{n^2}{n^6+4}$$

$$\therefore \left|\frac{n}{n^3+2i}\right| = \sqrt{\frac{n^2}{n^6+4}} = \frac{n}{\sqrt{n^6+4}}$$

$$\leq \frac{n}{\sqrt{n^6}} \qquad (n^6+4 \geq n^6)$$

$$= \frac{1}{n^2}.$$ So we have absolute convergence.

<u>#12 (p15)</u> If $n^2+z^2=0$, then no convergence. This means

$z^2 = -n^2$, or equivalently, $z = \pm ni$. $\quad (n \geq 0)$

For other values of $z \in \mathbb{C}$ we have absolute convergence:

$$\left|\frac{1}{n^2+z^2}\right|^2 = \frac{1}{(n^2+z^2)(n^2+\bar{z}^2)}$$

$$= \frac{1}{n^4 + n^2(\bar{z}^2+z^2) + |z|^4}$$

$$= \frac{1}{n^4 + 2n^2 \operatorname{Re} z + |z|^4} .$$

~~and this tends~~ Therefore

$$\left|\frac{1}{n^2+z^2}\right| = \frac{1}{\sqrt{n^4 + 2n^2 \operatorname{Re} z + |z|^4}} \sim \frac{1}{n^2} \text{ for } n \text{ large.}$$

Therefore we have absolute convergence (for $z \neq 0, \pm i, \pm 2i \ldots$)