Solutions to

Midterm #2
Mathematics 3150–2, Summer 2006
Department of Mathematics, University of Utah
June 26, 2006

Name: _____
Student ID Number: _____

* This is a closed-book, closed-notes examination.
* This exam begins at 11:00 a.m. and ends at 12:00 p.m. sharp.
* This exam is made up of 2 questions for a total of 20 points.
* Write your answers clearly. **If you show merely a numerical answer, then you are likely to receive zero partial credit.** So show and explain your work.
* **Confine your work to this worksheet.** You may use both sides of the paper. There is also an extra sheet of paper per problem for you to write on if you wish.
* There is a formula sheet in the very back of this worksheet which you may use at will.

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1. (10 points) Solve, explicitly, the heat equation,

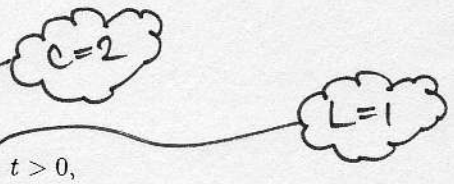
$$\frac{\partial u}{\partial t}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t), \qquad 0 < x < 1,\ t > 0,$$
$$u(x,0) = x,\ \frac{\partial u}{\partial t}(x,0) = 0, \qquad 0 < x < 1,$$
$$u(0,t) = u(1,t) = 0, \qquad t > 0.$$

$f(x) = x \longrightarrow b_n = 2\int_0^1 x \sin(n\pi x)\,dx = \frac{2}{n\pi}(-1)^{n+1}.$

$g(x) = 0 \longrightarrow b_n^* = 0$

$\therefore \quad u(x,t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi x)\,e^{-n^2\pi^2 t}.$

2. (10 points) Consider the following the wave equation,



$$\frac{\partial^2 u}{\partial t^2}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t), \qquad 0 < x < 1, \; t > 0,$$
$$u(x,0) = x, \; \frac{\partial u}{\partial t}(x,0) = 0, \qquad 0 < x < 1,$$
$$u(0,t) = u(1,t) = 0, \qquad t > 0.$$

$f(x) = x \longrightarrow b_n = \frac{2}{n\pi}(-1)^{n+1}$

$g(x) = 0 \longrightarrow b_n^* = 0$

$$u(x,t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(n\pi x)\cos(2n\pi t) \; .$$