Math 3150-2, Summer 2006
Solutions to Quiz 3

Note: $c=2$, $a=b=1$, $f(x,y)=32$.

$$\Rightarrow \lambda_{m,n} = 2\pi\sqrt{m^2+n^2}$$

$$A_{m,n} = 4\int_0^1\int_0^1 32 \sin(m\pi x)\sin(n\pi y)\,dx\,dy$$

$$= 128 \int_0^1 \sin(m\pi x)\,dx \cdot \int_0^1 \sin(n\pi y)\,dy$$

$$= \frac{128}{mn\pi^2}\left[1-\cos m\pi\right]\left[1-\cos n\pi\right]$$

$$\therefore \quad u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{128}{\pi^2 mn}(1-\cos m\pi)(1-\cos n\pi) \times \sin(m\pi x)\sin(n\pi y)\, e^{-4\pi^2(m^2+n^2)t}$$

~~$u(x,y,t)$~~

$$= g(x,t)\,g(y,t), \quad \text{where}$$

$$g(x,t) = \frac{\sqrt{128}}{\pi}\sum_{m=1}^{\infty}\frac{1-\cos m\pi}{m}\sin(m\pi x)\, e^{-4\pi^2 m^2 t}.$$