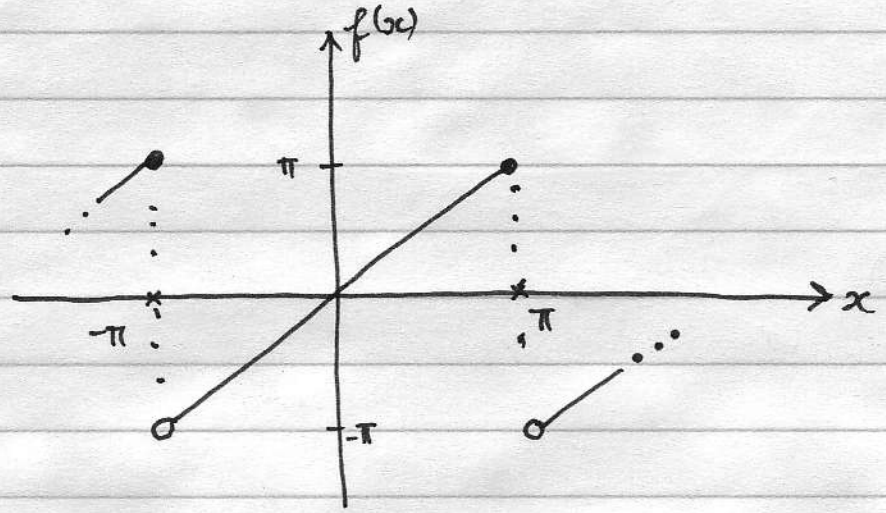
Math 3150-2
Solutions to Quiz #1

① 

② $f(\pi+) = -\pi \qquad f(\pi-) = +\pi$

③ f is an odd function; therefore $a_0 = a_1 = a_2 = \cdots = 0$.

For all $n \geq 1$,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)\,dx$$

$$= \frac{1}{\pi}\left[-\frac{x}{n}\cos(nx)\Big|_{-\pi}^{\pi} + \underbrace{\int_{-\pi}^{\pi}\frac{1}{n}\cos(nx)\,dx}_{0}\right]$$

$$= \frac{-1}{\pi n} x \quad x\cos(nx)\Big|_{-\pi}^{\pi}$$

$$= -\frac{1}{\pi n} \times \left[\pi\cos(n\pi) - (-\pi)\cos(-n\pi)\right]$$

$$= \frac{2}{n}\cos(n\pi) = \frac{2}{n} \times (-1)^{n+1}.$$

$u = x$; $v' = \sin(nx)$

$u' = 1$; $v = -\frac{1}{n}\cos(nx)$.

Therefore,

$$f(x) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cos(nx), \quad \text{for} \quad -\pi < x < \pi.$$

↑

N.B.: No π.