Math 3150-2, Summer '06
Solutions to Quiz 5

(a) Let $g(x) = e^{-x^2}$; we know: $\hat{g}(w) = e^{-w^2}$.

So, $\hat{f}(w) = i(\hat{g})'(w) = i(e^{-w^2})' = -2iwe^{-w^2}$.

(b) $$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} x e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} x \cos \omega x \, dx - \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} x \sin \omega x \, dx.$$

$\int_{-1}^{1} x \cos \omega x \, dx = 0$ because $x \cos \omega x$ is odd.

$$\int_{-1}^{1} x \sin \omega x \, dx = -\frac{1}{\omega} x \cos \omega x \Big|_{-1}^{1} + \frac{1}{\omega} \int_{-1}^{1} \cos \omega x \, dx$$

$u = x$, $v' = \sin \omega x$
$u' = 1$, $v = -\frac{1}{\omega} \cos \omega x$

$$= -\frac{1}{\omega}\left[\cos \omega + \cos(-\omega)\right] + \frac{1}{\omega^2} \sin \omega x \Big|_{-1}^{1}$$

$$= -\frac{2}{\omega} \cos \omega + \frac{2}{\omega^2} \sin \omega.$$

$$\hat{f}(\omega) = -\frac{i}{\sqrt{2\pi}} \left[-\frac{2}{\omega} \cos \omega + \frac{2}{\omega^2} \sin \omega \right].$$