# Formula Sheet for Midterm 1
# Math 3080-1, Spring 2007, The University of Utah

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Date: January 31, 2007
**Materials not covered here will not be tested**
Pages: 2 (not including title page)

**Testing for a mean**
$\boldsymbol{H_0: \mu = \mu_0}$ **versus** $\boldsymbol{H_a: \mu > \mu_0}$

- Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ and/or $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$.
- Reject when $Z > z_\alpha$ if $\sigma$ known and the population is normal, or if $n$ is large.
- Reject when $T > t_{\alpha,n-1}$ if $\sigma$ unknown, but the population is normal, or if $n$ is large.
- $P$-value $= 1 - \Phi(Z)$ or the corresponding formula for a $t$-test, depending on which is used.

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**Testing for a proportion**
$\boldsymbol{H_0: p = p_0}$ **versus** $\boldsymbol{H_a: p > p_0}$

- Test statistic: $Z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$.
- Reject when $Z > z_\alpha$ if $np_0$ and $n(1-p_0)$ are both at least 10.
- For smaller samples, reject if $\hat{p} > c$, and find $c$ such that $\mathrm{P}_{H_0}\{\hat{p} > c\} = \alpha$ from a binomial table.

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**Two-sample mean testing**
$\boldsymbol{H_0: \mu_1 - \mu_2 = \Delta_0}$ **versus** $\boldsymbol{H_a: \mu_1 - \mu_2 > \Delta_0}$

- Test statistic: $Z = \dfrac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}}$.
- Reject with $Z > z_\alpha$.
- $P$-value $= 1 - \Phi(Z)$.

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**Two-sample mean testing (large samples)**
$\boldsymbol{H_0: \mu_1 - \mu_2 = \Delta_0}$ **versus** $\boldsymbol{H_a: \mu_1 - \mu_2 > \Delta_0}$

- Test statistic: $Z = \dfrac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}}$.
- Reject with $Z > z_\alpha$.
- $P$-value $= 1 - \Phi(Z)$.

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**Two-sample statistics for** $\boldsymbol{\mu_1 - \mu_2}$
**(normal populations)**

- Test statistic for $H_0: \mu_1 - \mu_2 = \Delta_0$ versus $H_a: \mu_1 - \mu_2 > \Delta_0$:

$$T = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}}.$$

- Reject when $T > t_{\alpha,\nu}$, where

$$\nu = \frac{\left(\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}\right)^2}{\dfrac{(S_1^2/m)^2}{m-1} + \dfrac{(S_2^2/n)^2}{n-1}}$$

[round down to the nearest integer].

- $P$-value similar to the one-sample tests.
- CI for $\mu_1 - \mu_2$: $\bar{X} - \bar{Y} \pm t_{\alpha/2,\nu}\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}$.

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**Two-sample CI for** $\boldsymbol{\mu_1 - \mu_2}$

- If $\sigma$'s known, then $\bar{X} - \bar{Y} \pm z_{\alpha/2}\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}$
- If $\sigma$'s unknown but $n, m$ large, then $\bar{X} - \bar{Y} \pm z_{\alpha/2}\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}$.

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**Paired data**

- Test statistic for $H_0: \mu_D = \mu_0$ versus $H_a: \mu_D > \Delta_0$: $T = \dfrac{\bar{D} - \Delta_0}{S_D/\sqrt{n}}$.
- Reject when $T > t_{\alpha,n-1}$ (normal populations, or large samples)
- $P$-value similar to the one-sample tests.
- CI for $\mu_D$: $\bar{D} \pm t_{\alpha/2,n-1}\dfrac{S_D}{\sqrt{n}}$ (normal populations, or large samples).

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**Two-sample proportions**

- Test statistic for $H_0: p_1 - p_2 = 0$ versus $H_a: p_1 - p_2 > 0$ is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{m} + \dfrac{1}{n}\right)}},$$

where $\hat{p} = \dfrac{X+Y}{m+n} = \dfrac{m}{m+n}\hat{p}_1 + \dfrac{n}{m+n}\hat{p}_2$.

- Reject when $Z > z_\alpha$ ($n, m$ large).
- $P$-value similar to the one-sample problem.
- CI for $p_1 - p_2$:

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n}}$$

($n, m$ large).

**Two variances**

- Test statistic for $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_a: \sigma_1^2 > \sigma_2^2$: $F = \dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$.
- Reject if $F > F_{\alpha,m-1,n-1}$.
- $P$-value = the upper tail of $F_{n,m}$.
- $F_{1-\alpha,\nu_1,\nu_2} = \dfrac{1}{F_{\alpha,\nu_2,\nu_1}}$.
- CI for $\dfrac{\sigma_1^2}{\sigma_2^2}$: from $\dfrac{S_1^2}{F_{\alpha/2,n,m}\ S_2^2}$ to $\dfrac{S_1^2}{F_{1-(\alpha/2),n,m}\ S_2^2}$.