Math 3070-1, Fall 2009
Solutions to Homework 7

6.2. (a) Remember that independent normals add to a normal random variable; their means add; and their variances add. Therefore,

$$T \stackrel{\mathcal{D}}{=} N(500\,,121).$$

(b) We are asked to compute $P\{T > 520\}$. So we transform 520 grams into standard units:

$$\frac{520-500}{\sqrt{121}} = 1.81818181818;$$

The answer is the area to the right of 1.81818181818 under the standard-normal curve; i.e.,

$$P\{T > 520\} \simeq 1 - 0.9656 = 0.0344.$$

(c) We expect each toy to weigh 500 grams; therefore, 24 toys weigh a total of $24 \times 500 = 12,000$ grams [or 12 kilograms]. This and the 200 grams [for the box] together yield a total weight of $12,200$ grams [or 12.2 kilos]. The variances add: Each toy has variance 121 grams; therefore the total variance—or give and take squared—is $24 \times 121 = 2,904$ grams squared [or 2.904 kilogram squared].

(d) The total weight of the box is 200 + the sum of 24 independent normals. Therefore, its distribution is $N(12200\,,2904)$. In particular, the probability that the total weight is more than 12300 grams is the probability that a standard normal is at least

$$\frac{12300-12200}{\sqrt{2904}} \simeq 1.86$$

that probability is $1 - 0.9686 = 0.0314$ [the answer!!].

6.4. (a) Since $\bar{X}$ is $N(2\,,0.04)$,

$$\begin{aligned}
P\left\{1.9 < \bar{X} < 2.1\right\} &= P\left\{\frac{1.9-2}{\sqrt{0.04}} < N(0\,,1) < \frac{2.1-2}{\sqrt{0.04}}\right\} \\
&= P\left\{-0.1 < N(0\,,1) < 0.1\right\} \\
&\simeq 0.08.
\end{aligned}$$

(b) Since $\bar{X}$ is distributed as $N(2\,,4/n)$,

$$\begin{aligned}
P\left\{1.9 < \bar{X} < 2.1\right\} &= P\left\{\frac{1.9-2}{\sqrt{4/n}} < N(0\,,1) < \frac{2.1-2}{\sqrt{4/n}}\right\} \\
&= P\left\{-0.05\sqrt{n} < N(0\,,1) < 0.05\sqrt{n}\right\}.
\end{aligned}$$

In order for this probability to be 0.9, the area to the left of $0.05\sqrt{n}$—under the $N(0\,,1)$ distribution—must be 0.95. The normal table tells us that $0.05\sqrt{n} = 1.645$. Solve to obtain:

$$n = \left(\frac{1.645}{0.05}\right)^2 \simeq 1,082.41.$$

Therefore, n has to be around 1082 or 1083.

6.12. You need to draw a picture [as discussed in the lectures] in order to follow this.

(a) We want to use a χ^2_{10} table; the area to the left of a is 0.05; and the area to the left of b is 0.95. Therefore, $a = 3.94$ and $b = 18.307$.

(b) We want to use a χ^2_{15} table; the area to the left of a is 0.05; and the area to the left of b is 0.95. Therefore, $a = 7.261$ and $b = 24.996$.

(c) We want to use a χ^2_{10} table; the area to the left of a is 0.025; and the area to the left of b is 0.975. Therefore, $a = 3.247$ and $b = 20.483$.

(d) We want to use a χ^2_{20} table; the area to the left of a is 0.025; and the area to the left of b is 0.975. Therefore, $a = 9.591$ and $b = 34.170$.

6.16. According to Theorem 6.4, $(n-1)S^2/\sigma^2$ is distributed according to χ^2_{n-1}.

(a) Therefore,

$$\begin{aligned} P\{S < 0.0522\} \simeq P\left\{S^2 < 0.003\right\} &= P\left\{\frac{(n-1)S^2}{\sigma^2} < \frac{13 \times 0.00272484}{0.001764}\right\} \\ &= P\left\{\chi^2_{12} < 20.081\right\} \simeq 0.95. \end{aligned}$$

(b) Therefore,

$$\begin{aligned} P\{S > 0.0556\} \simeq P\left\{S^2 > 0.003\right\} &= P\left\{\frac{(n-1)S^2}{\sigma^2} > \frac{13 \times 0.00309136}{0.001764}\right\} \\ &= P\left\{\chi^2_{12} > 22.782\right\}, \end{aligned}$$

and this is between 0.05 and 0.025.

6.20. (a) $F_{3,20}(0.05) = 3.10$.

(b) $F_{3,20}(0.01) = 4.94$.

(c) $F_{4,30}(0.05) = 2.69$.

(d) $F_{4,30}(0.01) = 4.02$.