1. (a) $\boxed{\text{5 points}}$ Find the radian measure of the central of a circle of radius 80 cm that intercepts an arc of length 160 cm.

$s = r\theta$ $\quad r = 80$ cm

$\theta = \frac{s}{r}$ $\quad s = 160$

$\theta = \frac{160}{80}$ radians $= \boxed{2 \text{ radians}}$

(b) A circular power saw of radius 10 centimeters rotates at 5000 revolutions per minute.

i. $\boxed{\text{3 points}}$ Find the angular speed of the saw blade in radians per minute.

$\theta = 5000 \times 2\pi$ rad $= 10{,}000\pi$ rad

$t = 1$ min

$\therefore \omega = \frac{\theta}{t} = \boxed{10{,}000\pi \text{ rad/s}}$

ii. $\boxed{\text{2 points}}$ Find the linear speed of the saw blade in meters per minute.

[1 meter = 100 centimeters]

$v = r\omega$, $r = 10$ cm

$\therefore v = 10 \times 10{,}000\pi = 100{,}000\pi$ cm/s

$= \boxed{1000\pi \text{ meters/s}}$

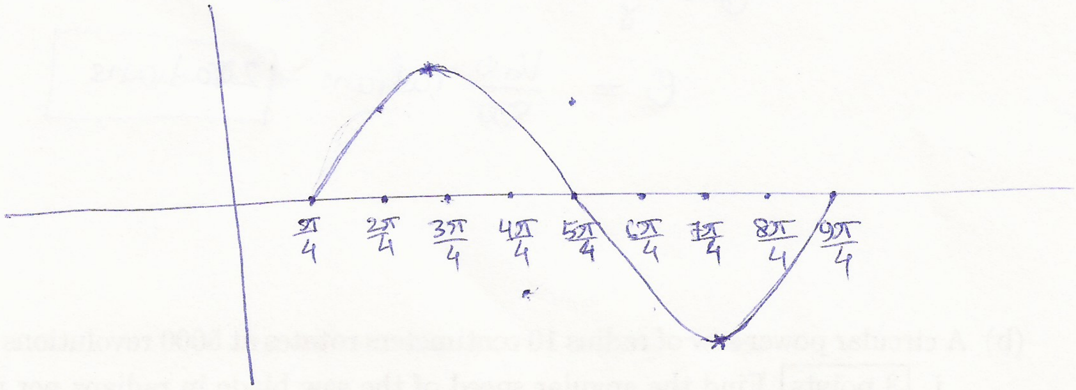
2. Sketch the graph of the following functions.

(a) $\boxed{\text{5 points}}$ $y = \sin\left(x - \frac{\pi}{4}\right)$

$x - \frac{\pi}{4} = 0 \Rightarrow x = \frac{\pi}{4}$, $x - \frac{\pi}{4} = 2\pi \Rightarrow x = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

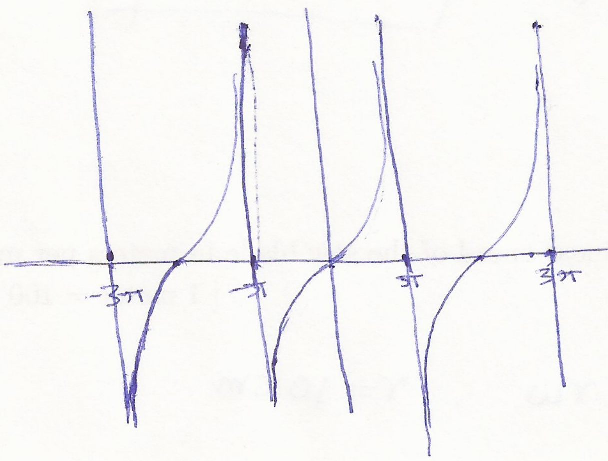
Period $= 2\pi$, Interval of one cycle $\left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$; $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$; $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$; $\left[\frac{5\pi}{4}, \frac{7\pi}{4}\right]$; $\left[\frac{7\pi}{4}, \frac{9\pi}{4}\right]$

$\frac{2\pi}{4} = \frac{\pi}{2}$



(b) $\boxed{\text{5 points}}$ $y = \tan\frac{x}{2}$

$\frac{x}{2} = -\frac{\pi}{2} \Rightarrow x = -\pi$, $\frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

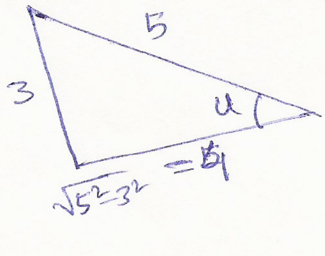


3. Evaluate

(a) $\boxed{\text{5 points}}$ $\tan\left(\arcsin\left(-\frac{3}{5}\right)\right)$

$u = \arcsin\left(-\frac{3}{4}\right)$, $-\frac{\pi}{2} \le u \le \frac{\pi}{2}$, i.e. u is in I or IV-th Quadrant

$\sin u = -\frac{3}{4} < 0$, $\Rightarrow u$ is in IV Quadrant.



$\therefore \tan\left(\arcsin\left(-\frac{3}{5}\right)\right)$

$= \tan u$

$= \boxed{-\frac{3}{4}}$ ($\because$ u in is IV Quadrant $\tan u < 0$)

This problem is missing another piece of information "θ is in Quadrant IV"

(b) $\boxed{\text{5 points}}$ If $\tan\theta = -\frac{3}{4}$, find the value of the rest of the trigonometric functions.

$\sin\theta = -\frac{3}{5}$

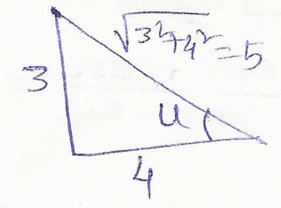
$\cos\theta = \frac{4}{5}$

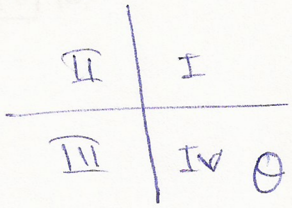
$\tan\theta = -\frac{3}{4}$

$\cot\theta = -\frac{4}{3}$

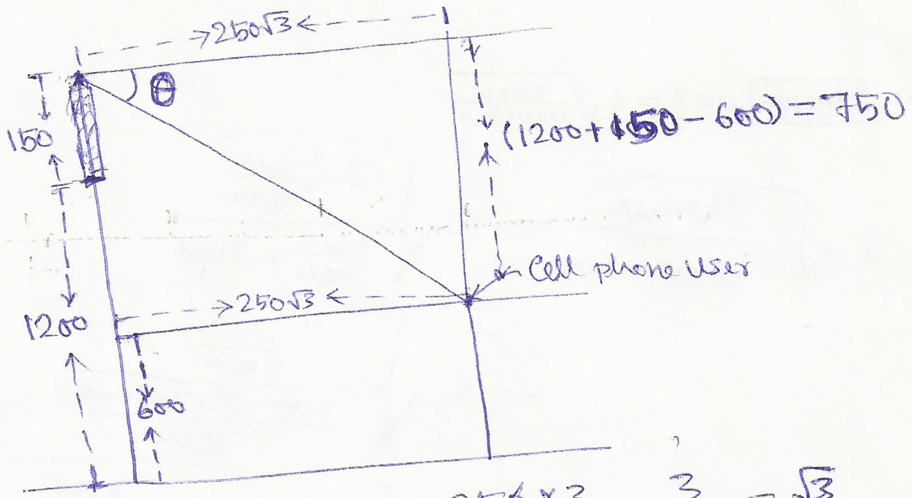
$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{5}{3}$





4. $\boxed{\text{10 points}}$ A cellular telephone tower that is 150 meters tall is placed on the top of a mountain that is 1200 meters above the sea level. What is the angle of depression from the top of the tower to a cell phone user who is $250\sqrt{3}$ horizontal meters away and 600 meters above the sea level?



$\therefore \tan\theta = \frac{750}{250\sqrt{3}} = \frac{\cancel{250}\times 3}{\cancel{250}\times\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore \theta = \boxed{60^\circ}$

5. Verify the following identities.

(a) $\boxed{\text{5 points}}$ $\frac{\cos t+\cos 3t}{\sin 3t-\sin t} = \cot t$

$$\frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \frac{\cos\left(\frac{t+3t}{2}\right)\cos\left(\frac{t-3t}{2}\right)}{\cos\left(\frac{t+3t}{2}\right)\sin\left(\frac{3t-t}{2}\right)}$$

$$= \frac{\cancel{\cos 2t}\cos(-t)}{\cancel{\cos 2t}\sin t}$$

$$= \frac{\cos t}{\sin t} \qquad \left[\because \cos \text{ is an even function } \cos(-t) = \cos t\right]$$

$$= \cot t$$

(b) $\boxed{\text{5 points}}$ $\sec\frac{u}{2} = \pm\sqrt{\frac{2\tan u}{\tan u+\sin u}}$

$$\pm\sqrt{\frac{2\tan u}{\tan u + \sin u}} = \pm\sqrt{\frac{2\frac{\sin u}{\cos u}}{\frac{\sin u}{\cos u} + \frac{\sin u}{1}}}$$

$$= \pm\sqrt{\frac{\frac{2\sin u}{\cos u}}{\frac{\sin u + \cos u \sin u}{\cos u}}}$$

$$= \pm\sqrt{\frac{2\cancel{\sin u}}{\cancel{\cos u}} \times \frac{\cancel{\cos u}}{\cancel{\sin u}(1+\cos u)}}$$

$$= \pm\sqrt{\frac{2}{1+\cos u}}$$

$$= \pm\sqrt{\frac{1}{\frac{1+\cos u}{2}}}$$

$$= \pm\sqrt{\frac{1}{\cos^2\frac{u}{2}}}$$

$$= \pm\sqrt{\sec^2\frac{u}{2}} = \pm\sec\frac{u}{2}$$

6. Solve the following trigonometric equations. Write the *general solutions* for each of the problem.

(a) $\boxed{\text{5 points}}$ $\tan 2x - 2\cos x = 0$

$$\tan 2x - 2\cos x = 0$$

$$\frac{\sin 2x}{\cos 2x} - 2\cos x = 0$$

$$\Rightarrow \sin 2x - 2\cos x \cos 2x = 0$$

$$\Rightarrow 2\sin x \cos x - 2\cos x \cos 2x = 0$$

$$\Rightarrow 2\cos x(\sin x - \cos 2x) = 0$$

$$\Rightarrow 2\cos x = 0, \quad \Big| \quad \sin x - \cos 2x = 0$$

$$\Rightarrow \cos x = 0, \quad \Big| \quad \Rightarrow \sin x - (1 - 2\sin^2 x) = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \Big| \quad \Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \Big| \quad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \Big| \quad x = \frac{3\pi}{2}$$

$$\therefore x = \boxed{2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}}$$

$$x = \boxed{2n\pi + \frac{\pi}{6}}, x = \boxed{2n\pi + \frac{5\pi}{6}}, x = \boxed{2n\pi + \frac{3\pi}{2}}$$

(b) $\boxed{\text{5 points}}$ $\sec^2 x + \tan x - 3 = 0$

$$\sec^2 x + \tan x - 3 = 0$$

$$1 + \tan^2 x + \tan x - 3 = 0$$

$$\Rightarrow \tan^2 x + \tan x - 2 = 0$$

$$\Rightarrow (\tan x + 2)(\tan x - 1) = 0$$

$$\Rightarrow \tan x = -2 \quad \Big| \quad \tan x = 1$$

$$x = \arctan(-2) \quad \Big| \quad x = \frac{\pi}{4}$$

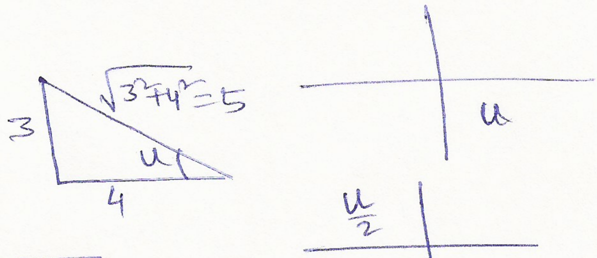
$$\therefore x = \boxed{n\pi + \arctan(-2)} \quad \Big| \quad \therefore x = \boxed{n\pi + \frac{\pi}{4}}$$

7. $\boxed{\text{10 points}}$ Given $\tan u = -\frac{3}{4}$, $\frac{3\pi}{2} < u < 2\pi$, find the exact value of $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, and $\tan\frac{u}{2}$.

$\because \frac{3\pi}{2} < u < 2\pi$, u is in Quadrant IV, $\therefore$

$\frac{3\pi}{2} < u < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{u}{2} < \pi$

$\therefore$ $\frac{u}{2}$ is in Quadrant II



$\therefore$ $\sin\frac{u}{2} > 0$, $\cos\frac{u}{2} < 0$, $\tan\frac{u}{2} < 0$

$\therefore$ $\sin\frac{u}{2} = +\sqrt{\frac{1-\cos u}{2}}$, $\cos\frac{u}{2} = -\sqrt{\frac{1+\cos u}{2}}$, $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$

$\cos u = \frac{4}{5}$, $\sin u = -\frac{3}{5}$ ($\because$ u is in Quadrant IV, $\sin u < 0$ & $\cos u > 0$)

$\therefore$ $\sin\frac{u}{2} = +\sqrt{\frac{1-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \boxed{\sqrt{\frac{1}{10}}}$

$\cos\frac{u}{2} = -\sqrt{\frac{1+\frac{4}{5}}{2}} = -\sqrt{\frac{\frac{9}{5}}{2}} = -\sqrt{\frac{9}{10}} = \boxed{-\frac{3}{\sqrt{10}}}$

$\tan\frac{u}{2} = \frac{1-\frac{4}{5}}{-\frac{3}{5}} = \frac{\frac{1}{5}}{-\frac{3}{5}} = \frac{1}{\cancel{5}}\times\left(-\frac{\cancel{5}}{3}\right) = \boxed{-\frac{1}{3}}$

8. (a) $\boxed{\text{5 points}}$ Solve the triangle(s) which has $a = 1, b = \sqrt{3}, A = 30°$. If two triangle exist, find both.

$\sin B = \frac{b}{a}\sin A = \frac{\sqrt{3}}{1}\sin 30° = \sqrt{3}\cdot\frac{1}{2} = \frac{\sqrt{3}}{2}$

$\therefore B = 60° > A$

$\therefore$ There are two triangles.

$B_1 = 60°$

$C = 180° - 30° - 60°$
$= 90°$

$\therefore \frac{c}{\sin C} = \frac{a}{\sin A}$

$\Rightarrow c = a\cdot\frac{\sin C}{\sin A}$

$= 1\cdot\frac{\sin 90°}{\sin 30°}$

$= 1\cdot\frac{1}{\frac{1}{2}} = 2$

$\therefore$ $a = 1$, $b = \sqrt{3}$, $c = 2$,
$A = 30°$, $B = 60°$, $C = 90°$

$B_2 = 180° - 60° = 120°$

$C = 180° - 30° - 120° = 30°$

$\therefore$ $c = a\cdot\frac{\sin C}{\sin A}$

$= 1\cdot\frac{\sin 30°}{\sin 30°}$

$= 1\cdot\frac{\frac{1}{2}}{\frac{1}{2}}$

$= 1$

$\therefore$ $a = 1$, $b = \sqrt{3}$, $c = 1$
$A = 30°$, $B = 120°$, $C = 30°$

(b) $\boxed{\text{5 points}}$ Solve the triangle which has $a = 2, b = \sqrt{3},$ and $c = 1$.

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(\sqrt{3})^2+1^2-2^2}{2\cdot\sqrt{3}\cdot 1} = \frac{3+1-4}{2\sqrt{3}} = \frac{0}{2\sqrt{3}} = 0$$

$\therefore A = 90^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b}{a}\sin A = \frac{\sqrt{3}}{2}\cdot \sin 90^\circ$$

$$= \frac{\sqrt{3}}{2}\cdot 1$$

$\therefore \sin B = \frac{\sqrt{3}}{2}$

$\Rightarrow B = 60^\circ$

$\therefore C = 180^\circ - 90^\circ - 60^\circ = 30^\circ$

9. (a) $\boxed{\text{5 points}}$ Find the component form of the vector $\mathbf{v}$ which has $||\mathbf{v}|| = \frac{7}{2}$ and direction angle $\theta = 150^\circ$.

$$\vec{v} = \|\vec{v}\|\langle\cos\theta, \sin\theta\rangle$$

$$= \frac{7}{2}\langle\cos 150^\circ, \sin 150^\circ\rangle$$

$$= \frac{7}{2}\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\right\rangle$$

$$= \boxed{\left\langle -\frac{7\sqrt{3}}{4}, \frac{7}{4}\right\rangle}$$

(b) $\boxed{\text{5 points}}$ Find the angle between the two vectors $\mathbf{u} = 2\mathbf{i}$ and $\mathbf{v} = \mathbf{i} + \sqrt{3}\mathbf{j}$.

$$\vec{u}\cdot\vec{v} = 2\cdot 1 + 0\cdot\sqrt{3} = 2 > 0$$

$$\therefore \theta = \arccos\left(\left|\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}\right|\right)$$

$$\|\vec{u}\| = \sqrt{2^2+0^2} = 2$$

$$\|\vec{v}\| = \sqrt{1^2+(\sqrt{3})^2} = 2$$

$$\therefore \theta = \arccos\left(\left|\frac{\cancel{2}^{1}}{2\cdot\cancel{2}_{1}}\right|\right) = \arccos\left(\frac{1}{2}\right) = \boxed{60^\circ}$$

Sorry! Here I missed the vectors, $\bar{u} = \langle -1, -2 \rangle$ & $\bar{v} = \langle 1, -1 \rangle$

10. (a) $\boxed{\text{5 points}}$ Find the projection of of **u** onto **v**. Then write **u** as the sum of two orthogonal vectors, one of which is proj $_{\mathbf{v}}\mathbf{u}$.

$$\text{Proj}_{\bar{v}}\bar{u} = \left(\frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\right)\bar{v}$$

$$\bar{u}\cdot\bar{v} = -1(1) + (-2)(-1) = -1+2 = 1$$

$$\|\bar{v}\| = \sqrt{1^2+(-1)^2} = \sqrt{2}$$

$$\therefore \text{Proj}_{\bar{v}}\bar{u} = \left(\frac{1}{(\sqrt{2})^2}\right)\langle 1,-1\rangle = \frac{1}{2}\langle 1,-1\rangle = \boxed{\left\langle \frac{1}{2}, -\frac{1}{2}\right\rangle}$$

$$\therefore \bar{u} - \text{Proj}_{\bar{v}}\bar{u} = \langle -1,-2\rangle - \left\langle \frac{1}{2}, -\frac{1}{2}\right\rangle = \left\langle -1-\frac{1}{2}, -2+\frac{1}{2}\right\rangle$$

$$= \left\langle -\frac{3}{2}, -\frac{3}{2}\right\rangle$$

$$\therefore \bar{u} = \text{Proj}_{\bar{v}}\bar{u} + (\bar{u} - \text{Proj}_{\bar{v}}\bar{u})$$

$$= \left\langle \frac{1}{2}, -\frac{1}{2}\right\rangle + \left\langle -\frac{3}{2}, -\frac{3}{2}\right\rangle$$

(b) $\boxed{\text{5 points}}$ Use DeMoivre's theorem to find $2(\sqrt{3}+i)^7$.

$$|\sqrt{3}+i| = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$$

$$\theta = \arctan\left(\left|\frac{1}{\sqrt{3}}\right|\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\therefore \sqrt{3}+i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$\therefore 2(\sqrt{3}+i)^7 = 2\left[2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^7$$

$$= 2\cdot 2^7\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$= 2^8\left(\cos\left(\pi+\frac{\pi}{6}\right) + i\sin\left(\pi+\frac{\pi}{6}\right)\right)$$

$$= 256\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$$

$$= 256\left(\frac{-\sqrt{3}-i}{2}\right)$$

$$= 128(-\sqrt{3}-i) = \boxed{-128\sqrt{3} - 128i}$$