Soln ①

Plan A:

$$A = P\left(1+\frac{APR}{n}\right)^{(nY)}$$

$A = \$120,000$, $APRY = 5\% = 0.05$

$n = 1$, $Y = 30$

$$P = \frac{A}{\left(1+\frac{APR}{n}\right)^{nY}}$$

$$= \frac{\$120,000}{\left(1+\frac{0.05}{1}\right)^{(1\times 30)}}$$

$$= \frac{\$120,000}{(1.05)^{30}}$$

$$= \$27765.29$$

Plan B:

$$A = P \times e^{(APR \times Y)}, \quad APR = 4.8\% = 0.048$$

$$P = \frac{A}{e^{(APR\times Y)}} = \frac{\$120,000}{e^{(0.048\times 30)}} = \frac{\$120,000}{e^{1.44}} = \$28431.33$$

~~$26115.62~~

Plan C:

$$A = PMT \times \frac{\left[\left(1+\frac{APR}{n}\right)^{(nY)}-1\right]}{\left(\frac{APR}{n}\right)}$$

$$PMT = \frac{A}{\frac{\left[\left(1+\frac{APR}{n}\right)^{(nY)}-1\right]}{\left(\frac{APR}{n}\right)}}$$

In this case $n = 12$, Since monthly payment, APR $= 3\% = 0.03$

$$\therefore PMT = \frac{\$120,000}{\frac{\left[\left(1+\frac{0.03}{12}\right)^{(12\times30)}-1\right]}{\frac{0.03}{12}}} = \frac{\$120,000}{\frac{\left[(1.0025)^{360}-1\right]}{0.0025}}$$

$$= \frac{\$120,000}{582.7369}$$

$$= \$205.92$$

Sol'n ②

$$A = PMT \times \frac{\left[\left(1+\frac{APR}{n}\right)^{(ny)}-1\right]}{\left(\frac{APR}{n}\right)}$$

$A = \$30,000$, $\therefore PMT = \$500$, $n = 12$, APR $= 5\% = 0.05$

$$30,000 = 500 \times \frac{\left[\left(1+\frac{0.05}{12}\right)^{12y}-1\right]}{\frac{0.05}{12}}$$

$$30,000 = 500 \times \frac{\left[(1.00417)^{12y}-1\right]}{0.00417}$$

$$\frac{60}{\frac{30,000}{500}} \times 0.00417 = \frac{500}{500} \times 0.00417 \times \frac{\left[(1.00417)^{12y}-1\right]}{0.00417}$$

$$0.2502 = (1.00417)^{12y} - 1$$

$$1 + 0.2502 = (1.00417)^{12y} - 1 + 1$$

$$1.2502 = (1.00417)^{12y}$$

$$\log_{10} 1.2502 = \log_{10} (1.00417)^{12y}$$

$$\log_{10} 1.2502 = 12y \times \log_{10} 1.00417 \qquad \left[\because \log_{10} a^x = x \times \log_{10} a\right]$$

$$12Y = \frac{\log_{10} 1.2502}{\log_{10} 1.00417} = 53.662$$

$$Y = \frac{53.662}{12} = 4.47$$

∴ You need 4.47 years.

Sol^n ③

For more money, the annual interest on the accumulated balance by $200,000 (for more detail look in your text book, Page: 233, Example 4).

6% of accumulated balance = $200,000

i.e. 6% × accu. balance = $200,000

$$\text{accumulated balance} = \frac{\$200,000}{6\%} = \$3333333.33$$

∴ $3333334 will be good enough.

$A = \$3333334$, $n = 12$, $Y = 40$, $APR = 6\% = 0.06$

$PMT = ?$

$$A = PMT \times \frac{\left[\left(1 + \frac{APR}{n}\right)^{(nY)} - 1\right]}{\left(\frac{APR}{n}\right)}$$

$$PMT = \frac{A}{\frac{\left[\left(1 + \frac{APR}{n}\right)^{nY} - 1\right]}{\left(\frac{APR}{n}\right)}}$$

$$= \frac{\$333334}{\frac{\left[\left(1+\frac{0.06}{12}\right)^{(12\times40)}-1\right]}{\frac{0.06}{12}}}$$

$$= \frac{\$333334}{\frac{\left[(1.005)^{480}-1\right]}{0.005}}$$

$$= \frac{\$333334}{1091.4907}$$

$$= \$1673.79$$

$\therefore$ (a) $\therefore$ PMT = \$1673.79

(b) Total deposit in part (a) $= \$1673.79 \times 40 \times 12$
$= \$803419.2$

Next Part:

$P = \$803419.2$
$APR = 4\% = 0.04$
$Y = 40$
$A = P \times e^{(APR \times Y)} = \$803419.2 \times e^{(0.04 \times 40)}$
$= \$3979361.848$

Sol'n (4)

(a) Interest = 2% of $2000 = $\frac{2}{100} \times 2000 = \$40$

Fee = 20% of $2000 = $\frac{20}{100} \times \$2000 = \$400$

Total = $\$40 + \$400 = \$440$ after first month

(b) Each month $440,

So, in a full year your pay = $\$440 \times 12$

$= \$5280$

(c) $$PMT = \frac{P \times \left(\frac{APR}{n}\right)}{\left[1 - \left(1 + \frac{APR}{n}\right)^{(-ny)}\right]}$$

$P = \$2000$, $APR = 10\% = 0.1$, $Y = 3$, $n = 12$

$$\therefore PMT = \frac{\$2000 \times \frac{0.1}{12}}{\left[1 - \left(1 + \frac{0.1}{12}\right)^{(-12\times3)}\right]} = \frac{\$2000 \times 0.00833}{0.25817}$$

$$= \$64.53$$

$\frac{\$64.53}{\$440} = 0.15 = 15\%$, therefore in case of Lamar you just pay 15% of what you were paying to car-title lender per month.

Soln. (5)

$$Q = Q_0 \times 2^{\frac{t}{T_{double}}}$$

$T_{double} = 3$ months

$$Q = 3Q_0$$

$$\therefore 3Q_0 = Q_0 \times 2^{\frac{t}{3}}$$

$$\Rightarrow \frac{3Q_0}{Q_0} = \frac{Q_0}{Q_0} \times 2^{\frac{t}{3}}$$

$$\Rightarrow 3 = 2^{\frac{t}{3}}$$

$$\Rightarrow \log_{10} 3 = \log_{10} 2^{\frac{t}{3}}$$

$$\log_{10} 3 = \frac{t}{3} \times \log_{10} 2$$

$$\frac{t}{3} = \frac{\log_{10} 3}{\log_{10} 2}$$

$$t = 3 \times \frac{\log_{10} 3}{\log_{10} 2} = 4.75$$

(a) $\therefore$ **4.75 months**

(b) 2 years $= 2 \times 12$ months $= 24$ months

$$Q = 500 \times 2^{\frac{24}{3}} = 500 \times 2^8 = \boxed{128000}$$

Sol'n ⑥

(a) $Q = Q_0 \times (1+r)^t$

$Q_0 = 300\,mg,\ Q = 120\,mg,\ t = 6-2 = 4\,hr$

$120 = 300 \times (1+r)^4$

$\Rightarrow \frac{120}{300} = \frac{300}{300} \times (1+r)^4$

$0.4 = (1+r)^4$

$\Rightarrow \sqrt[4]{0.4} = \sqrt[4]{(1+r)^4}$

$0.7953 = 1+r$

$r = \pm 0.7953 - 1 = -0.2047 \approx -0.20 = -20\%$

∴ rate of decrease is 20%

(b) $T_{half} = -\frac{\log_{10} 2}{\log_{10}(1-0.20)} = -\frac{\log_{10} 2}{\log_{10} 0.80} = 3.11$

∴ Half-life is 3.11 hours.

⑦

(a) Independent variable = time = t (secs)
Dependent variable = height of snow = h (secs).

(b) Initial value of h = 3

Rate of change = $\frac{5}{2} = 2.5$

Depen. Variable = Initial value + rate of change × Indepen. Variable

∴ $\boxed{h = 3 + 2.5t}$

(c) $h = 21$

$21 = 3 + 2.5t$

$21 - 3 = 3 + 2.5t - 3$

$18 = 2.5t$

$t = \frac{18}{2.5} = 7.2$

∴ It took 7.2 hours.

Sub. (8)

Independent variable = time = t (say)

Dependent " = Price = P (say)

Initial value = \$1200

Rate of change = − \$75 (negative sign since the value decreasing as the time increasy)

∴ $P = 1200 + (-75) \times t$

i.e. $\boxed{P = 1200 - 75t}$

$P = 0$, $0 = 1200 - 75t$

$-1200 = -75t$

$t = \frac{-1200}{-75} = 16$

∴ after 16 years its value will be \$0.

Sol'n (9)

Independent variable = Weight = w (say)

Dependent " = Speed = s (say)

Two pairs of information is $(0, 50)$ and $(20, 40)$

$\therefore$ rate of fall = Slope = $\frac{50-40}{0-20} = \frac{10}{-20} = -0.5$

Let b be the initial value

$\therefore \quad s = b + (-0.5) \times w$

$\therefore \quad s = b - 0.5w$

Putting $(0, 50)$ in the above equation we have

$50 = b - 0.5 \times 0$

$50 = b$

$\therefore \quad \boxed{s = 50 - 0.5w}$

$s = 0, \quad 0 = 50 - 0.5w$

$-50 = -0.5w$

$w = \frac{-50}{-0.5} = \boxed{100}$

$\therefore$ at 100 tons of cargo, the speed is 0 miles per hour.

Sol: (ii)

$$Q = Q_0 * (1+r)^t$$

$r = -8\% = -0.08$ (negative sign, since it is decay)

$Q_0 = 1500$

$\therefore \quad Q = 1500 \times (1-0.08)^t$

$$Q = 1500 \times (0.92)^t$$

In will face extinction, when its population is 30 i.e. $Q = 30$

$$\therefore 30 = 1500 \times (0.92)^t$$

$$\frac{30}{1500} = (0.92)^t$$

$$\frac{1}{50} = (0.92)^t$$

$$0.02 = (0.92)^t$$

$$\log_{10} 0.02 = \log_{10} (0.92)^t$$

$$\log_{10} 0.02 = t \times \log_{10} 0.92$$

$$\frac{\log_{10} 0.02}{\log_{10} 0.92} = t$$

$$t = 46.92$$

$\therefore$ it will face extinction after 46.92 years.

Soln. (11)

(a) $T_{half} = 4.5$ billion years.

(a) $Q = Q_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T_{half}}} = Q_0 \times (0.5)^{\frac{t}{4.5}}$

$Q = 65\%$ of $Q_0 = 0.65 \times Q_0$

$\therefore 0.65 \times Q_0 = Q_0 \times (0.5)^{\frac{t}{4.5}}$

$0.65 = (0.5)^{\frac{t}{4.5}}$

$\log_{10} 0.65 = \log_{10} (0.5)^{\frac{t}{4.5}}$

$\log_{10} 0.65 = \frac{t}{4.5} \times \log_{10} (0.5)$

$\frac{\log_{10} 0.65}{\log_{10} 0.5} = \frac{t}{4.5}$

$\frac{\log_{10} 0.65}{\log_{10} 0.5} \times 4.5 = t$

$2.8 = t$

$\therefore$ The rock is 2.8 billion years old.

(b) $Q = 45\%$ of $Q_0 = 0.45 \times Q_0$

$$0.45 \times Q_0 = Q_0 \times (0.5)^{\frac{t}{4.5}}$$

$$\log_{10} 0.45 = \log_{10} (0.5)^{\frac{t}{4.5}}$$

$$\log_{10} 0.45 = \frac{t}{4.5} \times \log_{10} 0.5$$

$$\frac{\log_{10} 0.45}{\log_{10} 0.5} = \frac{t}{4.5}$$

$$\frac{\log_{10} 0.45}{\log_{10} 0.5} \times 4.5 = t$$

$$1.2 = t$$

∴ This rock is 1.2 billion years old.

** Important Note: In P.No (11), the other ~~part of~~ piece of information about Lead (In Part(a) and Part(b) both) has nothing to do with your calculation. It's just to confuse you more! 😉