Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. **Show all your work.**

Name: _Solution Key_   ID: U ___00___

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	18	15	10	10	12	15	100
Score:									

The formulas below are provided for your convenience

Savings Plan: $A = \text{PMT} \times \dfrac{\left[\left(1+\frac{\text{APR}}{n}\right)^{(nY)} - 1\right]}{\left(\frac{\text{APR}}{n}\right)}$

Loan Payment: $\text{PMT} = \dfrac{P \times \left(\frac{\text{APR}}{n}\right)}{\left[1 - \left(1+\frac{\text{APR}}{n}\right)^{(-nY)}\right]}$

Logarithm: $\log_{10} a^x = x \times \log_{10} a$

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1. (a) (2 points) You deposit \$100 in an account with an APR of 8% and continuous compounding. How much will you have after 10 years?

$$A = \$100 \times e^{(0.08 \times 10)} = \$100 \times e^{0.8} = \$100 \times 2.22554$$

$$\boxed{= \$222.55}$$

(b) (8 points) You deposit \$1000 in an account that pays an APR of 7% compounded annually. How long will it take for your balance to reach \$100,000?

$P = \$1000$, $\text{APR} = 7\% = 0.07$, $n = 1$,

$A = \$100,000$, $Y = ?$

$A = P \times \left(1 + \dfrac{\text{APR}}{n}\right)^{(nY)}$

$100,000 = 1000 \times (1 + 0.07)^{(1 \times Y)}$

$\dfrac{100,000}{1000} = (1.07)^Y$

$100 = (1.07)^Y$

$\log_{10} 100 = \log_{10}(1.07)^Y = Y \times \log_{10} 1.07$

$\dfrac{\log_{10} 100}{\log_{10} 1.07} = Y$ $\boxed{68.06 = Y}$ $\therefore$ 68.06 years

2. A savings account pays an annual percentage rate (APR) of 3.5% compounded quarterly.

(a) (5 points) Find the annual percentage yield (APY) on this account.

$$P = \$100, \quad APR = 3.5\% = 0.035, \quad n = 4, \quad Y = 1$$

$$A = \$100 \times \left(1 + \frac{0.035}{4}\right)^{(4 \times 1)} = \$100 \times 1.03546 = \$103.55$$

$$\therefore APY = \frac{\$103.55 - \$100}{\$100} = \frac{\$3.55}{\$100} = \boxed{3.55\%}$$

(b) (5 points) You decided that you would like to make a regular quarterly deposits to this account since you would like to have $500,000 when you retire in 35 years. How much should your quarterly deposits be in order to accomplish your goal?

$$A = \$500,000, \quad n = 4, \quad Y = 35, \quad APR = 3.5\% = 0.035$$

$$A = PMT \times \frac{\left[\left(1 + \frac{APR}{n}\right)^{(ny)} - 1\right]}{\left(\frac{APR}{n}\right)}$$

Solve for PMT

$$PMT = \frac{A}{\frac{\left[\left(1 + \frac{APR}{n}\right)^{(ny)} - 1\right]}{\left(\frac{APR}{n}\right)}} = \frac{\$500,000}{\frac{\left[\left(1 + \frac{0.035}{4}\right)^{(4 \times 35)} - 1\right]}{\left(\frac{0.035}{4}\right)}}$$

$$= \frac{\$500,000}{\frac{2.3861}{0.00875}}$$

$$= \boxed{\$1833.54}$$

3. You have found that you are eligible for a 30 year house loan with annual interest rate (APR) of 6.25%, compounded monthly.

(a) (4 points) If you take out this loan for $220,000, what will your monthly payment be?

$$APR = 6.25\% = 0.0625, \quad Y = 30, \quad n = 12, \quad P = \$220,000$$

$$PMT = \frac{P \times \left(\frac{APR}{n}\right)}{\left[1 - \left(1 + \frac{APR}{n}\right)^{(-ny)}\right]} = \frac{\$220,000 \times \left(\frac{0.0625}{12}\right)}{\left[1 - \left(1 + \frac{0.0625}{12}\right)^{(-12 \times 30)}\right]} = \frac{\$1145.833}{0.84590}$$

$$= \boxed{\$1354.57}$$

(b) (4 points) How much will you pay in interest (in $ terms) over the life of the loan if you take out this loan for $220,000?

$$\text{Total payment} = \$1354.57 \times 12 \times 30 = \$487,645.2$$

$$\text{Interest} = \$487,645.2 - \$220,000 = \boxed{\$267,645.2}$$

(c) (10 points) If you decide instead to get a 20-year loan at the same rate for the same amount, what would your monthly payment be and how much would you save (in dollars) in interest (if you decided to take a 20 year loan instead of 30 year loan)?

$$PMT = \frac{\$220,000 \times \frac{0.0625}{12}}{\left[1-\left(1+\frac{0.0625}{12}\right)^{(-12\times20)}\right]} = \frac{\$1145.833}{0.7126} = \$1,607.96$$

Interest paid $= \$1,607.96 \times 12 \times 20 - \$220,000 = \$165,910.4$

You would save $\boxed{(\$267,645.2 - \$165,910.4) = \$101,734.8}$ in interest.

4. You can afford monthly payment of $500. If current mortgage rates are 9% for 30-year fixed rate loan.

(a) (10 points) What loan principal can you afford? $PMT = \$500, Y = 30, n = 12 \rightarrow APR = 9\% = 0.09$

$$PMT = \frac{P \times \left(\frac{APR}{n}\right)}{\left[1-\left(1+\frac{APR}{n}\right)^{(-ny)}\right]}$$

$$P = \frac{PMT}{\frac{\left(\frac{APR}{n}\right)}{\left[1-\left(1+\frac{APR}{n}\right)^{(-ny)}\right]}} = \frac{\$500}{\frac{\left(\frac{0.09}{12}\right)}{\left[1-\left(1+\frac{0.09}{12}\right)^{(-30\times12)}\right]}}$$

$$= \frac{\$500}{\frac{0.0075}{0.9321148}}$$

$$= \frac{\$500 \times 0.9321148}{0.0075}$$

$$\boxed{= \$62,140.99}$$

(b) (5 points) If you are required to make a 20% down payment and you have the cash on hand to do it, how expensive a home can you afford?

$\$62,140.99 + 20\% \text{ of } \$62,140.99$

$$\boxed{= \$74,569.19}$$

5. Suppose your pet dog weighed 2.5 pounds at birth and weighed 15 pounds after one year.

   (a) (2 points) Identify the Independent and Dependent variable.

   Independent variable → time ('t)

   Dependent   ,,   → weight ('W)

   (b) (5 points) Find a linear model which describes this situation.

   $(0, 2.5) ; (1, 15)$

   slope = rate of change = $\frac{15 - 2.5}{1 - 0} = \frac{12.5}{1} = 12.5$

   ∴ $W = b + 12.5 \times t$

   $2.5 = b + 12.5 \times 0$ $2.5 = b$

   ∴ $\boxed{W = 2.5 + 12.5 \times t}$

   (c) (3 points) Use your model to predict your dog's weight at 5 and 10 years of age.

   Weight at 5th year $= 2.5 + 12.5 \times 5 = 2.5 + 62.5 = \boxed{65 \text{ pounds}}$

   Weight at 10th year $= 2.5 + 12.5 \times 10 = 2.5 + 125 = \boxed{127.5 \text{ pounds}}$

6. You can purchase a motorcycle for $6500 or lease it for a down payment of $200 and $150 per month.

   (a) (1 point) Which model is this problem related to, linear or exponential?

   Linear, since fixed growth $150 per month

   (b) (5 points) Find a function that describes how the cost of the lease depends on time.

   time → 't, Cost → C

   $\boxed{C = 200 + 150 \times t}$

   (c) (4 points) Use your function to find how long can you lease the motorcycle before you've paid more than its purchase price.

   $6500 = 200 + 150 \times t$

   $6500 - 200 = 150 \times t$

   $6300 = 150 \times t$ $\frac{6300}{150} = t$

   $\boxed{42 = t}$

   ∴ You can lease for 42 months.

7. A certain medication breaks down in the human body (decreases) at a rate of 12% per hour.

(a) (3 points) Find the approximate half-life.

$$T_{half} \approx \frac{70}{12} = \boxed{5.83 \text{ hours}}$$

(b) (3 points) Find the exact half-life of that medication in your bloodstream.

$$T_{half} = -\frac{\log_{10} 2}{\log_{10}(1+r)} \qquad , \quad r = -12\% = -0.12$$

$$= -\frac{\log_{10} 2}{\log_{10}(1-0.12)} = -\frac{\log_{10} 2}{\log_{10} 0.88} = \frac{0.301030}{-0.055517} = \boxed{5.42 \text{ hours}}$$

(c) (6 points) If you took 500 mg of this medication at 2 pm, how much is left in your bloodstream at 9 pm?

2 pm to 9pm is 7 hours.

$$500 mg \times \left(\frac{1}{2}\right)^{\frac{7}{5.42}} = \cancel{500 mg} \quad \boxed{204.26 \text{ mg}}$$

8. (15 points) A toxic radioactive substance with a density of 3 milligrams per square centimeter is detected in the ventilating ducts of a nuclear processing building that was used 55 years ago. If the half-life of the substance is 20 years, what was the density of the substance when it was deposited 55 years ago?

$$Q = Q_0 \times \left(\frac{1}{2}\right)^{\frac{t}{T_{half}}}$$

$$T_{half} = 20 \text{ years}, \quad t = 55 \text{ years}, \quad Q = 3 \text{ milligram/cm}^2$$

$$Q_0 = ?$$

$$3 = Q_0 \times (0.5)^{\frac{55}{20}}$$

$$\frac{3}{(0.5)^{2.75}} = Q_0 \qquad \boxed{20.18 = Q_0}$$

∴ 55 years ago the density was $\boxed{20.18}$ milligram/cm^2