Math 2250-03

Mechanical Oscillations - Maple Project 2

Notes and assignment

Monday October 20

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Project Directions: This project deals with concepts discussed in Chapter 5. It would be best to work through this project with your book in hand. This project is adapted from Professor Gustafson's version. There are three parts to it, and each part has a few questions associated with it. You will be handing in a single Maple document containing your answers. Clearly label each of your answers if you want to receive the credit for them. Some parts may call for handwritten work - make your answers legible, either by leaving yourself some space in the document to (neatly) fill in by hand, or create the answers as text displays.

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Part 1 - Underdamped Oscillations

Consider the equation for a free damped linear oscillator

```
> m*diff(x(t), t,t) + c*diff(x(t),t) + k*x(t) = 0;
```

$$m\left(\frac{d^2}{dt^2}\,\mathrm{x}(t)\right) + c\left(\frac{d}{dt}\,\mathrm{x}(t)\right) + k\,\mathrm{x}(t) = 0$$

with initial conditions:

```
> x(0)=0, D(x)(0)=1;
```

$$\mathrm{x}(0) = 0,\ \mathrm{D}(x)(0) = 1$$

where, m, c and k are non-negative constants.

Exercise 1. Using the information given to you on page 327, if $m = 3$ and $c = 4$ find (by hand) a value for k that would provide the scenario of underdamping. How did you arrive at this conclusion? Next, solve the characteristic polynomial (by hand or in Maple). How does this value of k affect the roots of this polynomial, and thus the dynamics of the differential equation? Now, using Maple, display the exact solution x(t) for this equation.

Exercise 2. Plot the solution x(t) vs. time. You should get something that looks like Figure 5.4.9 on page 328. Now on the same graph, include the plot of the decaying exponential.

Exercise 3. From your plot found in Exercise 2, estimate the value of the pseudoperiod. You can click with the mouse on the graphic to print the cursor location in the left upper corner of the maple window. The coordinates printed are of the form (x,y). From this coordinate information, you can estimate the period by simple subtraction.

Next, find the exact pseudoperiod as given on page 328 of your book. Do theses two values agree?

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A little bit of sample code to help you on your way through Part 1 (as found on Prof. Gustafson's page...)

```
> restart:  #clear all old definitions from memory
> with(DEtools):    #load the DE library of commands
> with(plots):
> #Note:  in your code, use semicolons...  and use your numbers
#rather
> than letters for the mass, spring, and damping #constants.
> #Defining the differential equation
> de:= m*diff(x(t), t,t) + c*diff(x(t),t) + k*x = 0:
> #solving the characteristic polynomial can be done with
> solve(a*r^2+b*r+c=0,r):
> #Defining initial conditions
> ic:= x(0)=d, D(x)(0)=f:
> #Symbolically solve for x(t)
> p:=dsolve({de,ic}, x(t), method=laplace):
> #Capture the dsolve symbolic solution as a function of X(t)
> X:= unapply(rhs(p),t):
> #Plot the solution
> plot(X(t), t=0..5):
```

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Part 2 - Undamped Forced Oscillations

Consider the undamped (c=0) forced problem

```
> m*diff(x(t), t,t) + k*x(t) = 5*cos(omega*t);
```

$$m\,(\frac{d^2}{dt^2}\,\mathrm{x}(t)) + k\,\mathrm{x}(t) = 5\cos(\omega\,t)$$

with initial conditions

```
> x(0)=0, D(x)(0)=0;
```

$$\mathrm{x}(0) = 0,\ \mathrm{D}(x)(0) = 0$$

where m, k and w are non-negative constants. For this problem, assume that m=3 and k=4.5. Start by reading through pages 349-352 so you know where you are headed.

Exercise 4. Choose the forcing angular frequency to be 3 times larger than the natural angular frequency (see page 349) . Solve for x(t) using dsolve. Plot the solution on a suitable interval in order to show the global behavior of the solution x(t). You should get something that looks like Figure 5.6.2 on page 350.

Exercise 5. The solution x(t) is the sum of two functions, one of period 2Pi/w and the other of period 2Pi/w0. Using your solution to x(t) from Exercise 4 (and page 350 as a guide), what is the value of the exact period?

Exercise 6. Suggest a value for the forcing frequency so that the oscillations exhibit resonance (show how you arrive at this value). Make a graph of resonant behavior as in Figure 5.6.4 on page 352.

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Tips for Part 2- With a little modification, you can use the same code as in part 1.

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Part 3 - Resonance

Finally, we will consider a scenario with elements from Part 1 and Part 2. Consider a damped forced problem,

```
> m*diff(x(t), t,t) + c*diff(x(t),t) + k*x(t) = 5*cos(w*t);
```

$$m\,(\frac{d^2}{dt^2}\,\mathrm{x}(t)) + c\,(\frac{d}{dt}\,\mathrm{x}(t)) + k\,\mathrm{x}(t) = 5\cos(w\,t)$$

with initial conditions:

```
> x(0)=0, D(x)(0)=0;
```

$$x(0) = 0,\ D(x)(0) = 0$$

and now assume that $m = 3$ and $k = 45$.

Exercise 7. Consider the damping constants c=2, c=1, and c=1/2. Compute the amplitude function C(w) as found on page 357 of your text for these three equations, then plot three amplitude graphs on a single set of axes for w = 0 to 20. You should end up with three curves that look like that of Figure 5.6.9 on page 357.

Exercise 8. Using the mouse on your plot of this graph, find the values of w* and C* for the three separate cases, where C* is the maximum value of the amplitude function in your plot (you should end up with a table of six values). You will find that as c -> 0, C* gets larger and larger. What does this tell you about the model?

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Some sample code for Part 3

```
> #First, define your parameters, then
> F:=f:  m:=g:  k:=h:  c:='c':  w:='w':
> C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
> plot({C(w,c1), C(w,c2), C(w,c3)},w=0..20,color=black):
```

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