

(1)

Math 2250-3

HW due 10/29

$$5.3 \quad (3, 10, 14) \quad 21 \quad (22) \quad 24 \quad (29, 33, 37)$$

$$5.4 \quad (4, 5, 7) \quad 10, 12, (13, 18, 23)$$

$$5.5 \quad (3) \quad 4 \quad (12) \quad 13 \quad (19) \quad (34, 37, 43, 49) \quad 50, (52)$$

$$5.3(3) \quad y'' + 3y' - 10y = 0 \\ \text{for } y = e^{rx} \text{ need } r^2 + 3r - 10 = 0$$

$$(r+5)(r-2) = 0; \quad r = -5, 2 \\ y(x) = c_1 e^{2x} + c_2 e^{-5x}$$

$$5.10) \quad 5y^{(4)} + 3y^{(3)} = 0 \\ \text{for } y = e^{rx} \text{ get } 5r^4 + 3r^3 = 0$$

$$r^3(5r+3) = 0$$

$$r=0 \quad (\text{multiplicity 3}) \\ r=-\frac{3}{5}$$

$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-\frac{3}{5}x}$$

$$5.14) \quad y^{(4)} + 3y'' - 4y = 0$$

$$\text{for } y = e^{rx} \text{ get } r^4 + 3r^2 - 4 = 0$$

$$(r^2+4)(r^2-1) = 0; \quad r = \pm 2i, \pm i$$

$$y(x) = c_1 \cos 2x + c_2 \sin 2x + c_3 \cos x + c_4 \sin x$$

$$e^{(2i)x} = \cos 2x + i \sin 2x \\ e^{ix} = \cos x + i \sin x$$

$$21) \quad 9y'' + 6y' + 4y = 0$$

$$9r^2 + 6r + 4 = 0$$

$$r = \frac{-6 \pm \sqrt{36-144}}{18} = \frac{-6 \pm \sqrt{108}}{18} = \frac{-6 \pm 6\sqrt{3}i}{18}$$

$$= -\frac{1}{3} \pm \frac{1}{\sqrt{3}}i$$

$$e^{(-\frac{1}{3} + \frac{1}{\sqrt{3}}i)x} = e^{-\frac{1}{3}x} \left[\cos \frac{x}{\sqrt{3}} + i \sin \frac{x}{\sqrt{3}} \right]$$

$$y(x) = e^{-\frac{1}{3}x} \left[c_1 \cos \frac{x}{\sqrt{3}} + c_2 \sin \frac{x}{\sqrt{3}} \right]$$

$$y'(x) = e^{-\frac{1}{3}x} \left[-\frac{1}{3}(c_1 \cos \frac{x}{\sqrt{3}} + c_2 \sin \frac{x}{\sqrt{3}}) + -\frac{c_1}{\sqrt{3}} \sin \frac{x}{\sqrt{3}} + \frac{c_2}{\sqrt{3}} \cos \frac{x}{\sqrt{3}} \right]$$

$$y(0) = c_1 = 3$$

$$y'(0) = -\frac{1}{3}c_1 + \frac{1}{\sqrt{3}}c_2 = 14$$

$$c_1 = 3 \\ -1 + \frac{c_2}{\sqrt{3}} = 14$$

$$\frac{c_2}{\sqrt{3}} = 15, \quad c_2 = 15\sqrt{3}$$

$$y(x) = e^{-\frac{1}{3}x} \left[3 \cos \frac{x}{\sqrt{3}} + 15\sqrt{3} \sin \frac{x}{\sqrt{3}} \right]$$

$$29) \quad y^{(3)} + 27y = 0$$

$$r^3 + 27 = 0 \quad r = -3 \text{ is a root; } r+3 \text{ is a factor}$$

$$r+3 \quad \begin{array}{r} r^3 + \\ r^3 + 3r^2 \\ \hline -3r^2 + 27 \end{array}$$

$$\text{roots } r = \frac{3 \pm \sqrt{9-36}}{2} = \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$$

$$y(x) = e^{\frac{3}{2}x} \left[c_1 \cos \frac{3\sqrt{3}}{2}x + c_2 \sin \frac{3\sqrt{3}}{2}x \right]$$

$$+ c_3 e^{-\frac{3}{2}x}$$

$$e^{\left(\frac{3}{2} + \frac{3\sqrt{3}}{2}i\right)x} = e^{\frac{3}{2}x} \left[\cos \frac{3\sqrt{3}}{2}x + i \sin \frac{3\sqrt{3}}{2}x \right]$$

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37) $y^{(4)} - y^{(2)} = 0$
 characteristic poly $r^4 - r^2 = 0$
 $r^2(r-1) = 0 \quad e^x, 1, x, x^2$

$$\begin{aligned}y(x) &= c_1 + c_2 x + c_3 x^2 + c_4 e^x \\y' &= c_2 + 2c_3 x + c_4 e^x \\y'' &= 2c_3 + c_4 e^x \\y^{(4)} &= c_4 e^x\end{aligned}$$

$$\begin{aligned}y(0) &= c_1 + c_4 = 18 & c_1 = 11 \\y'(0) &= c_2 + c_4 = 12 & c_2 = 5 \\y''(0) &= 2c_3 + c_4 = 13 & c_3 = 3 \\y'''(0) &= c_4 = 7 & c_4 = 7\end{aligned}$$

$$\boxed{y(x) = 11 + 5x + 3x^2 + 7e^x}$$

33) $y^{(3)} + 3y'' - 54y = 0$
 $r^3 + 3r^2 - 54 = 0$ since $y = e^{3x}$ is soln, $r=3$ is root,
 $r-3$ is factor

$$\begin{array}{r} r^2 + 6r + 18 \\ \hline r-3 | r^3 + 3r^2 - 54 \\ \hline r^3 - 3r^2 \\ \hline 6r^2 - 18r \\ \hline 18r - 54 \\ \hline 0 \end{array} \quad \begin{aligned}r &= \frac{-6 \pm \sqrt{36-72}}{2} \\ &= -6 \pm 6i \\ &= -3 \pm 3i\end{aligned}$$

$$\boxed{y(x) = c_1 e^{3x} + e^{-3x} [c_2 \cos 3x + c_3 \sin 3x]}$$

b5.4

$$4. m = .25 \text{ kg}$$

$$k(.25) = 9 \text{ N}$$

$$k = 36 \text{ N/m}$$

$$x_0 = 1 \text{ m}$$

$$v_0 = -5 \text{ m/s}$$

$$.25 x'' + 36 x = 0$$

$$x'' + 144x = 0$$

$$x = e^{rt}$$

$$p(r) = r^2 + 144 = 0$$

$$r^2 = -144$$

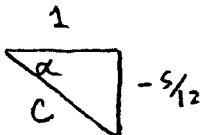
$$r = \pm 12i$$

$$e^{rt} = e^{\pm 12it} = \cos 12t \pm i \sin 12t$$

$$\text{so } x_H(t) = A \cos 12t + B \sin 12t$$

$$x(0) = 1 = A \quad A = 1$$

$$x'(0) = -5 = 12B; \quad B = -\frac{5}{12}$$



$$C = \sqrt{1 + \frac{25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12} \quad \text{amplitude}$$

$$\alpha = \tan^{-1}\left(-\frac{5}{12}\right) \quad \text{phase}$$

$$x(t) = \frac{13}{12} \cos(12t - \alpha) \approx 1.083 \cos(12t + 43^\circ)$$

$$\boxed{\text{period} = \frac{2\pi}{12} = \frac{\pi}{6} \text{ seconds}}$$

5. Pendulum eqtn

$$\text{is } \theta''(t) + \frac{g}{L} \theta = 0$$

$$\text{so } \omega_0 = \sqrt{\frac{g}{L}} \text{ rad/sec. ,}$$

$$\theta(t) = C \cos(\omega_0 t - \alpha)$$

$$\text{period} = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{L}{g}}$$

in this problem

$$\frac{P_1}{P_2} = \frac{2\pi \sqrt{\frac{L_1}{g_1}}}{2\pi \sqrt{\frac{L_2}{g_2}}} = \sqrt{\frac{L_1}{L_2}} \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{L_1}{L_2}} \sqrt{\frac{GM/R_2^2}{GM/R_1^2}} \\ = \sqrt{\frac{L_1}{L_2}} \frac{R_1}{R_2} \quad \checkmark$$

$$7. \frac{P_1}{P_2} = \frac{R_1}{R_2} \sqrt{\frac{L_1}{L_2}} \quad \text{unit don't matter (cancel in quotient)}$$

$$\begin{aligned} P_1 &= P_2 \\ \Rightarrow \frac{R_2}{R_1} &= \sqrt{\frac{L_1}{L_2}} \\ R_1 &= 3960 \\ L_1 &= 100.1 \\ L_2 &= 100.0 \end{aligned}$$

$$\begin{aligned} R_2 &= 3960 \sqrt{\frac{100.1}{100.0}} \\ &\approx 3961.98 \text{ miles} \\ &\begin{array}{r} 3961.979 \\ - 3960.0 \\ \hline 1.979 \end{array} \text{ miles} \end{aligned}$$

$$\begin{aligned} &\times 5280 \text{ ft/m} \\ &\approx 10,452 \text{ ft} \end{aligned}$$

$$(13) \begin{cases} m x'' + c x' + kx = 0 \\ 10 x'' + 9 x' + 2x = 0 \\ x(0) = 0 \\ x'(0) = 5 \end{cases}$$

$$x = e^{rt} : 10r^2 + 9r + 2 = 0 \\ (5r+2)(2r+1) = 0$$

$$r = -\frac{1}{2}, -\frac{2}{5} \text{ (overdamped)} \\ x(t) = c_1 e^{-\frac{1}{2}t} + c_2 e^{-\frac{2}{5}t} \\ x'(t) = -\frac{1}{2}c_1 e^{-\frac{1}{2}t} - \frac{2}{5}c_2 e^{-\frac{2}{5}t}$$

$$x(0) = 0 = c_1 + c_2$$

$$x'(0) = 5 = -\frac{1}{2}c_1 - \frac{2}{5}c_2$$

$$\begin{aligned} c_1 + c_2 &= 0 & | \cdot S \\ -5c_1 - 4c_2 &= 50 & | \cdot 1 \end{aligned}$$

$$\begin{cases} c_2 = 50 \\ c_1 = -50 \end{cases}$$

$$x(t) = -50e^{-\frac{1}{2}t} + 50e^{-\frac{2}{5}t}$$

$$x(t) = 50e^{-0.4t} [1 - e^{-1.1t}]$$

$$x(t) > 0 \quad t > 0$$

x_{\max} when $x'(t) = 0$

$$x'(t) = +25e^{-\frac{1}{2}t} - 20e^{-\frac{2}{5}t} \\ = e^{-0.4t} [-20 + 25e^{-0.1t}] = 0 : \begin{aligned} 25e^{-0.1t} &= 20 \\ e^{-0.1t} &= 0.8 \\ -0.1t &= \ln(0.8) \end{aligned}$$

$$x_{\max} = x(T)$$

$$T = \frac{\ln 0.8}{-0.1} \approx 2.23$$

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$$\begin{cases} 2x'' + 12x' + 50x = 0 \\ x(0) = 0 \\ v(0) = -8 \end{cases}$$

$$\text{where } 25e^{-0.1T} = 20; \quad e^{-0.1T} = \frac{5}{4}$$

$$so \quad x(T) = 50 \left(\frac{4}{5}\right)^4 \left(1 - \frac{4}{5}\right)$$

$$= \frac{50 \cdot 4^4}{5^5} = 10 \left(\frac{4}{5}\right)^4 \neq 4.096$$

$$x'' + 6x' + 25x = 0$$

$$r^2 + 6r + 25 = 0$$

$$(r+3)^2 + 16 = 0$$

$$(r+3-4i)(r+3+4i) = 0 \quad r = -3 \pm 4i$$

underdamped

$$e^{(-3+4i)t} = e^{-3t} (\cos 4t + i \sin 4t)$$

$$x_H(t) = e^{-3t} (A \cos 4t + B \sin 4t)$$

$$x_H(0) = 0 = A$$

$$x_H'(0) = -8 = -3A + 4B; \quad B = -2$$

$$x_H(t) = -2e^{-3t} \sin 4t$$

$$= e^{-3t} (-2 \sin 4t) = 2e^{-3t} \cos(4t + \frac{\pi}{2})$$

"triangle" $A=0, B=-2, C=2, D=-\pi/2$

requested form
 \downarrow
 \downarrow

$$\text{4.5.4 (23) a) } 100x'' + kx = 0$$

$$\omega_0 = \sqrt{\frac{k}{100}}$$

$$\text{told } f = \frac{\omega_0}{2\pi} = 80 \text{ cycles/min} = \frac{4}{3} \text{ cycles/sec.}$$

$$\omega_0 = \frac{8\pi}{3} \text{ rad/sec}$$

$$\left(\frac{8\pi}{3}\right)^2 = \omega_0^2 = \frac{k}{100}; \quad k = 100 \left(\frac{8\pi}{3}\right)^2 \approx 7018 \text{ lb/ft}$$

$$23 b) \frac{\omega_1}{2\pi} = \frac{78}{60} \text{ cycles/sec}$$

$$= 1.3 \text{ cycles/sec}$$

$$\omega_1 = 2.6\pi \text{ rad/sec.}$$

$$x'' + cx' + 70.18 = 0 \quad (\text{divided eqn by 100})$$

$$r^2 + cr + 70.18 = 0$$

$$r = \frac{-c \pm \sqrt{c^2 - 280.74}}{2}$$

$$2.6\pi = \omega_1 = \frac{\sqrt{280.74 - c^2}}{2}$$

$$4(2.6\pi)^2 = 280.74 - c^2$$

$$c^2 = 280.74 - 4(2.6\pi)^2$$

$$c \approx 3.724$$

$$P = \frac{c}{2} \approx 1.862$$

$$x(t) = e^{-1.862t} (C \cos(2.6\pi t - \alpha))$$

$$\text{solve } e^{-1.862t} = .01$$

$$4.5.5 \quad 3) y'' - y' - 6y = 2 \sin 3x$$

$$-6 \quad (y_p = A \cos 3x + B \sin 3x)$$

$$-1 \quad (y_p' = -3A \sin 3x + 3B \cos 3x)$$

$$1 \quad (y_p'' = -9A \cos 3x - 9B \sin 3x)$$

$$L(y_p) = \cos 3x [-9A - 3B - 6A] \\ + \sin 3x [-9B + 3A - 6B]$$

$$y_p = \frac{1}{39} \cos 3x - \frac{5}{39} \sin 3x$$

$$t = \frac{\ln(.01)}{-1.862} \approx 2.47 \text{ sec}$$

$$-9A - 3B = 0$$

$$-3A - 3B = 0$$

$$3A - 15B = 2$$

$$A = \begin{vmatrix} 0 & -1 \\ 2 & -15 \end{vmatrix} = \frac{2}{78} = \frac{1}{39}; \quad B = \frac{5}{39}$$

(5)

$$12) y''' + y' = 2 - \sin x$$

$$y_H: r^3 + r = 0$$

$$r(r^2 + 1) = 0$$

so both 2 & $\sin x$ are y_H 's.

so, for

$$L(y_p) = 2$$

$$\text{try } y_p = Ax$$

$$y_p' = A$$

$$y_p'' = y_p''' = 0$$

$$L(y_p) = A = 2$$

$$\boxed{y_p = 2x}$$

$$L(y_p) = -\sin x$$

$$\text{try } y_p = x(A \cos x + B \sin x)$$

$$\begin{aligned} \text{get } y_p''' + y_p' &= -2A \cos x - 2B \sin x = -\sin x \\ (\text{after a lot of work or MAPLE}) \end{aligned}$$

$$A = 0$$

$$B = \frac{1}{2}$$

$$\boxed{y_p = \frac{1}{2}x \sin x}$$

by superposition, for #12 $\boxed{y_p = 2x + \frac{1}{2}x \sin x}$

$$19) y^{(5)} + 2y^{(3)} + 2y'' = 3x^2 - 1$$

$$y_H: r^5 + 2r^3 + 2r^2 = 0$$

$$r^2(r^3 + 2r + 2) = 0$$

so 1, x are y_H 's

so try

$$y_p = x^2 [Ax^2 + Bx + C]$$

$$= Ax^4 + Bx^3 + Cx^2$$

$$y_p' = 4Ax^3 + 3Bx^2 + 2Cx$$

$$2(y_p'' = 12Ax^2 + 6Bx + 2C)$$

$$2(y_p''' = 24Ax + 6B)$$

$$0(y_p^{(4)} = 24A)$$

$$+ 1(y_p^{(5)} = 0)$$

$$L(y_p) = 24Ax^2 + (12B + 48A)x + (4C + 12B)1 = 3x^2 - 1$$

$$24A = 3; A = \frac{1}{8}$$

$$12B + 48A = 0; 12B + 6 = 0; B = -\frac{1}{2}$$

$$4C + 12B = -1; 4C - 6 = -1; 4C = 5; C = \frac{5}{4}$$

$$\boxed{y_p = \frac{1}{8}x^4 - \frac{1}{2}x^3 + \frac{5}{4}x^2}$$

$$34) \begin{cases} y'' + y = \cos x \\ y(0) = 1 \\ y'(0) = -1 \end{cases}$$

$$y_H(x) = c_1 \cos x + c_2 \sin x$$

$$1(y_p = x(A \cos x + B \sin x))$$

$$0(y_p' = x(-A \sin x + B \cos x) + A \cos x + B \sin x) + 1(y_p'' = x(-A \cos x - B \sin x) + 2(-A \sin x + B \cos x))$$

$$L(y_p) = -2A \sin x + 2B \cos x = \cos x$$

$$A = 0, B = \frac{1}{2}$$

$$y_p = \frac{1}{2}x \sin x$$

$$y = \frac{1}{2}x \sin x + c_1 \cos x + c_2 \sin x$$

$$y(0) = 1 = c_1$$

$$y'(0) = -1 = c_2$$

$$\boxed{y = \frac{1}{2}x \sin x + \cos x - \sin x}$$