Chapters 6,7,9 Review **Math 2250** **December 15, 2008**

Name:

1. Consider a predator-prey population consisting of foxes and rabbits living in a certain forest. Initially there are F_0 foxes and R_0 rabbits; after k months there are F_k foxes and R_k rabbits. Assume that the transition from each month to the next is described by the equations:

$$\begin{aligned} F_{k+1} &= .4F_k - .3R_k \\ R_{k+1} &= -rF_k + 1.2R_k \end{aligned}$$

The population vector satisfies $\overline{x}_{k+1} = A\overline{x}_k$, implying $\overline{x}_k = A^k\overline{x}_0$ where $\overline{x}_k = [F_k R_k]^T$.

a.) Describe what each of the terms on the right hand side of these two equations represents.

b.) Find the characteristic equation of the transition matrix and solve the eigenvalues.

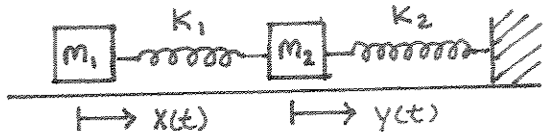
c.) If r=.5, what is the long term behavior of the system?

d.) If r=.4, what is the long term behavior of the system?

2. Apply the eigenvalue method to find a general solution of the following system:

$$\begin{aligned} x_1' &= 5x_1 - 9x_2 \\ x_2' &= 2x_1 - x_2 \end{aligned}$$

3. Consider the following configuration of a mass and spring system, with positive displacements from equilibrium measured to the RIGHT, as indicated:



a.) Derive the system of second order differential equations which corresponds to the above figure.

b.) Assume that in appropriate units $m_1 = 2$, $m_2 = 2$, $k_1 = 4$, $k_2 = 6$. Show that in this case the system above reduces to:

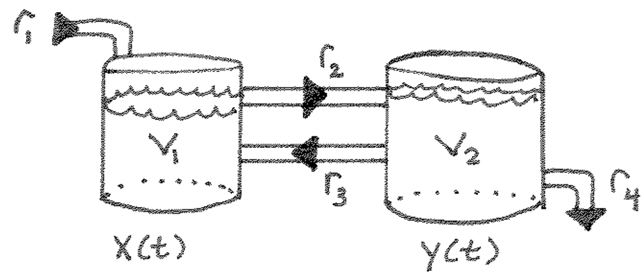
$$\begin{aligned} x'' &= -2x + 2y \\ y'' &= 2x - 5y \end{aligned}$$

c.) Find the general solution to this unforced system, and indicate the parts of the solution that contribute to the separate modes of frequency.

d.) Assuming that ω is not one of the natural frequencies for the problem in part c), find a particular solution to the forced system

$$
\begin{aligned}
x'' &= -2x + 2y + \cos(\omega t) \\
y'' &= 2x - 5y - \cos(\omega t)
\end{aligned}
$$

4. Consider the following 2 tank configuration. In tank one, there is a uniformly mixed volume of V1 gallons, and pounds of solute x(t). In tank two there is a mixed volume of V2 gallons and pounds of solute y(t). Water is pumped into tank one at a rate constant of r1 gallons/minute from an outside source, and this water has a constant solute concentration of c1 lbs./gal. Water is pumped from T1 to T2 at a constant rate of r2 gal/min, from T2 to T1 at a constant rate r3 gal/min, and out of the tank system at a constant rate r4 gal/min.



a) What conditions on the rates r1, r2, r3, r4 are necessary to guarantee that the volumes V1 and V2 remain constant in time?

b) Write down, but do not solve, the system of first order differential equations which governs the process described above.

5. Go over your homework. Go over examples we have done in class. The test is longer than the review and you will have one hour to complete it.

IDEAS:

- Characteristic equations
- eigenvalues / eigenvectors
- Diagonalization / Similar Matrices
- Transition Matrices
- Stochastic Matrices
- Principle of Superposition
- solutions of homogeneous/nonhomogeneous systems
- solutions of the form $\vec{x}' = A\vec{x}$
- solutions of the form $\vec{x}'' = A\vec{x}$
- applications of these systems
- mass-spring systems:
    - natural modes of oscillation
    - natural frequencies
    - constructing a system of DEs
- critical point behavior in the phase plane
- stability of critical points
- linear systems and linearization of almost linear systems
- Jacobian
- Bifurcation parameters
- Nonlinear mechanical systems

Chubby bunny.