2. Solve the following initial value problem:

$$2x^2y + x^3y' = 1, y(1) = 1$$

What is the form of this differential equation? What is the reason you picked the method you did in order to solve this problem?

$$x^3 y' + 2x^2 y = 1$$

$$y' + \frac{2}{x} y = \frac{1}{x^3}$$

Integrating factor is:

$$e^{\int \frac{2}{x} dx} = e^{(2 \ln x)} = x^2$$

multiplying the equation by the integrating factor

$$x^2 y' + 2xy = \frac{1}{x}$$

via the product rule..

$$\frac{d}{dx}(x^2 \cdot y) = \frac{1}{x}$$

$$\int \frac{d}{dx}(x^2 y) dx = \int \frac{1}{x} dx + C$$

$$x^2 y = \ln x + C$$

$$y(1) = 1 \Rightarrow 1 = \ln 1 + C$$

$$\Rightarrow C = 1$$

$$\underline{y(x) = \frac{1}{x^2} \ln x + \frac{1}{x^2}}$$

(Note that we didn't just use the formula from our book here. This is just to show where it comes from!)

4. Solve the following initial value problems:

a) $y' = -2y, y(0) = 10$

$$\int \frac{dy}{y} = \int -2\,dx + C$$

$$\ln y = -2x + C$$

$$y(x) = Ae^{-2x}$$

$$y(0) = 10 \Rightarrow A = 10$$

$$\underline{y(x) = 10e^{-2x}}$$

b) $y' = y^2 sin(x), y(0) = 1$

$$\int \frac{dy}{y^2} = \int \sin x\,dx + C$$

$$-\frac{1}{y} = -\cos x + C$$

$$y(x) = \frac{1}{c + \cos x}$$

$$y(0) = 1 \Rightarrow 1 = \frac{1}{c + \cos(0)}$$

$$\Rightarrow c = 1$$

$$\underline{y(x) = \frac{1}{1 + \cos x}}$$

c) $y' + 3x^2y = 6x^2, y(0) = 3$

$$y' = 3x^2(2 - y)$$

$$\int \frac{dy}{2-y} = \int 3x^2 dx + C$$

$$-\ln(2-y) = x^3 + C$$

$$\ln\left[\frac{1}{2-y}\right] = x^3 + C$$

$$\frac{1}{2-y} = c\exp(x^3)$$

$$2 - y = c\exp(-x^3)$$

$$y(x) = 2 + c\exp(-x^3)$$

$$y(0) = 3 \Rightarrow c = 1$$

$$\underline{y(x) = 2 + e^{-x^3}}$$

(note, you could also get this by using an integrating factor)

d) $y' = 6e^{2x-y}, y(0) = 0$

$$\int e^y dy = \int 6e^{2x} dx + C$$

$$e^y = 3e^{2x} + c$$

$$y(x) = \ln(3e^{2x} + c)$$

$$y(0) = 0 \Rightarrow \ln(3 + c) = 0$$

$$3 + c = 1$$

$$c = -2$$

$$\underline{y(x) = \ln(3e^{2x} - 2)}$$