

Name:

1. For each of the following systems of equations, find the augmented matrix corresponding to it. Then *either* reduce the matrix to the reduced row echelon form and use the result to solve the system, *or* reduce the matrix to the row echelon form and use backward substitution to solve the system.

(a)

$$2x_1 + 8x_2 + 3x_3 = 2$$

$$x_1 + 3x_2 + 2x_3 = 5$$

$$2x_1 + 7x_2 + 4x_3 = 8$$

(b)

$$3x_1 - 6x_2 + x_3 + 13x_4 = 15$$

$$3x_1 - 6x_2 + 3x_3 + 21x_4 = 21$$

$$2x_1 - 4x_2 + 5x_3 + 26x_4 = 23$$

2. Use Gauss-Jordan elimination to find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

3. Compute the determinants of each of the following matrices and use it to determine whether the matrix is singular or nonsingular (know what nonsingularity implies about a matrix!!):

(a)

$$\begin{pmatrix} 2 & -3 \\ 2 & 3 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 2 \\ 0 & 3 & 1 & 4 \\ 1 & 0 & 4 & 0 \end{pmatrix}$$

4. Consider the following linear system of equations

$$3x_1 + x_2 + 2x_3 = 1$$

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + 2x_2 + 2x_3 = 1$$

(a) Write down the corresponding matrix equation for the system above

(b) Compute the determinant of the 3×3 coefficient matrix. What does the value of the determinant tell you about the solution of the system? In particular, can you rule out if the system has one solution, no solution, or infinitely many solutions?

(c) Find the inverse of the coefficient matrix.

(d) Find the solution to the linear system.

5. Let A be the matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

(a) Show that this matrix is row equivalent to the 2×2 identity matrix provided that $ad - bc \neq 0$.

(b) What is the inverse of A?

6. Using cofactor expansion, find the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 5 & 0 & 6 & 0 \\ 0 & 7 & 0 & 8 \end{pmatrix}$$

7. Solve the linear system $Ax = b$ using Cramer's rule, where A is:

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 0 & 2 \\ -1 & -2 & 1 \end{pmatrix}$$

and b is:

$$\begin{pmatrix} -7 \\ 0 \\ 12 \end{pmatrix}$$

8. Find the solution subspace of the linear system $Ax = 0$, where

$A =$

$$\begin{pmatrix} 2 & 4 & -2 & 0 & -16 \\ 4 & 0 & 4 & -8 & 0 \end{pmatrix}$$

9. Consider the matrix below,

$$A = \begin{pmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 5 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{pmatrix} \quad (1)$$

with reduced row echelon form,

$$B = rref(A) = \begin{pmatrix} 1 & 0 & 3 & 0 & \frac{1}{2} \\ 0 & 1 & 2 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)$$

Find the basis for the solution space of the given homogeneous linear system. What is the dimension of the nullspace? What is the rank of the coefficient matrix?