

Investigating platelet motion towards vessel walls in the presence of red cells

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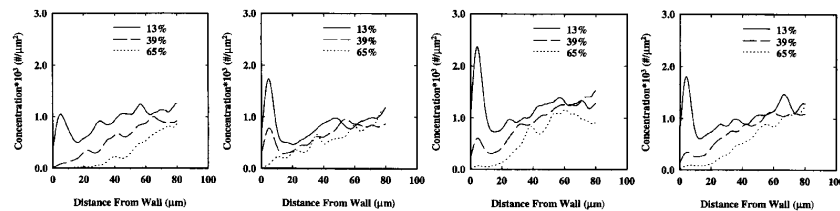
Abstract

It is a well established fact that platelets are highly concentrated near arteriole walls *in vivo*. This phenomenon depends on the deformability of red cells, hematocrit, and the size of platelets. However, it has proven difficult to quantify the effect these factors have on platelet motion towards the vessel wall.

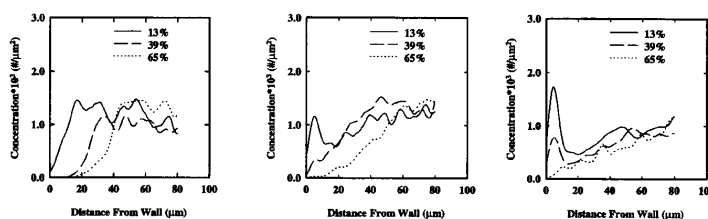
We use a lattice Boltzmann-immersed boundary method to solve the flow dynamics of red cells and platelets in a 2D vessel with no-slip boundary conditions. Red cells are treated as biconcave immersed boundary objects with isotropic Skalak membrane tension and an internal viscosity five times that of the surrounding plasma. We investigate radial platelet motion for both elliptical and spherical shaped platelets of various sizes. Using this method we can also analyze the influence of shear rate, hematocrit and red cell membrane properties.

Lateral platelet motion

Blood is a heterogenous medium composed primarily of red blood cells and plasma. Platelets only take up about 0.3% of blood by volume, yet have a non-uniform concentration that is highly localized near vessel walls where they are needed to repair vessel damage. Experiments by Eckstein *et al.* show that this concentration profile is highly sensitive to both shear rate and hematocrit.



The figure above demonstrates the effect of shear rate on lateral platelet motion (left to right: 250, 560, 800, 1220 s⁻¹).



This figure shows that hematocrit also influences lateral platelet motion (left to right: 0% 15% 40%).

Immersed Boundary Method

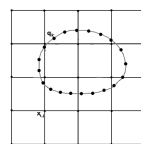
Immersed boundary method is a way of coupling fluid dynamics to the mechanics of some elastic boundary location within the fluid. Fluid lies on Eulerian grid (\mathbf{x}) and the boundary lives on a Lagrangian grid ($\mathbf{X}(q, t)$). An external force is imposed on the fluid by the presence of the immersed boundary object.

The force felt by fluid is related to the force felt by the boundary,

$$\mathbf{F}_f = \int \mathbf{F}_{IB} \delta(\mathbf{x} - \mathbf{X}(q, t)) dx,$$

and the boundary moves with the fluid velocity,

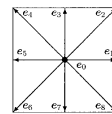
$$\frac{\partial \mathbf{X}}{\partial t}(q, t) = \mathbf{u}(\mathbf{X}, t) = \int \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(q, t)) dx.$$



Lattice Boltzmann Method

The lattice Boltzmann equations govern the behavior of the fluid in our simulations. These equations track particle distribution functions $f(\mathbf{x}, \mathbf{e}_i, t)$ that live at each Eulerian lattice node \mathbf{x} at time t . Each PDF is allowed to move along a set of discretized velocity vectors, \mathbf{e}_i . We choose the nine-velocity model:

$$\mathbf{e}_i = \begin{cases} (0, 0) & \text{for } i = 0 \\ (c \cos(\pi(i-1)/4), c \sin(\pi(i-1)/4)) & \text{for } i = 1, 3, 5, 7 \\ (c\sqrt{2} \cos(\pi(i-1)/4), c\sqrt{2} \sin(\pi(i-1)/4)) & \text{for } i = 2, 4, 6, 8, \end{cases}$$



where $c = \frac{h}{\delta t}$ is the particle speed. The equations governing the particle distribution functions in the presence of an external force are

$$f_i(\mathbf{x} + \mathbf{e}_i \delta t, t + \delta t) - f_i(\mathbf{x}, t) = \frac{1}{\tau} \left(f_i(\mathbf{x}, t) - f_i^{eq} \left(\rho, \mathbf{u} + \frac{\delta t \mathbf{F}_f}{2\rho} \right) \right) + \delta t w_i \left[\frac{(1 - \frac{1}{2\tau})(\mathbf{F}_f \cdot \mathbf{e}_i)}{c_s^2} - \frac{\frac{\delta t + \rho \delta^2}{\delta t \tau} (\mathbf{v} \mathbf{F}_f^T + \mathbf{F}_f \mathbf{v}^T) : (\mathbf{e}_i \mathbf{e}_i^T - c_s^2 \mathbf{I})}{2c_s^4} \right]$$

where τ is the relaxation parameter and f^{eq} is the equilibrium distribution. The exact form of f^{eq} depends on lattice geometry. For our nine-velocity model it is:

$$f_i^{eq}(\rho, \mathbf{u}) = \rho w_i \left(1 + \frac{\mathbf{e}_i \cdot \mathbf{u}}{c_s} + \frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{2c_s^2} - \frac{\mathbf{u} \cdot \mathbf{u}}{2c_s^2} \right)$$

where $c_s = \frac{c}{\sqrt{3}}$ is the speed of sound and the weights, w_i , are

$$w_i = \begin{cases} 4/9 & \text{for } i = 0 \\ 1/9 & \text{for } i = 1, 3, 5, 7 \\ 1/36 & \text{for } i = 2, 4, 6, 8. \end{cases}$$

The macroscopic quantities $\rho(\mathbf{x}, t)$ and $\mathbf{u}(\mathbf{x}, t)$ are the density and macroscopic fluid velocity. These quantities can be calculated from the particle distribution functions by

$$\rho(\mathbf{x}, t) = \sum_i f_i(\mathbf{x}, t) \text{ and } \rho(\mathbf{x}, t) \mathbf{u}(\mathbf{x}, t) = \sum_i f_i(\mathbf{x}, t) \mathbf{e}_i.$$

In the limit that the lattice spacing approaches zero and the particle speed c approaches infinity, the lattice Boltzmann equations approximate the Navier Stokes equations where the fluid pressure is

$$p(\mathbf{x}, t) = c_s^2 \rho = \frac{c^2}{3} \rho$$

and fluid viscosity is

$$\mu = \rho c_s^2 \delta t \left(\tau - \frac{1}{2} \right).$$

Modeling Cells

To model the behavior of one-dimensional cellular membranes, we use the following force equation

$$\mathbf{F}_{IB} = \frac{\partial \mathbf{T}}{\partial l} = \frac{\partial}{\partial l} (T \mathbf{t} + q \mathbf{n}).$$

Tension is both tangent to plane (due to stretching) and normal to plane (due to bending). For in plane tension we use a one-dimensional reduction of the Skalak membrane law:

$$T_{iso}^{sk} = G(\lambda^2 - 1)(1 - C\lambda^2(\lambda^2 + 1)),$$

where $\lambda = \frac{ds}{ds_0}$ is the principal stretch ratio. The equation for q is given by

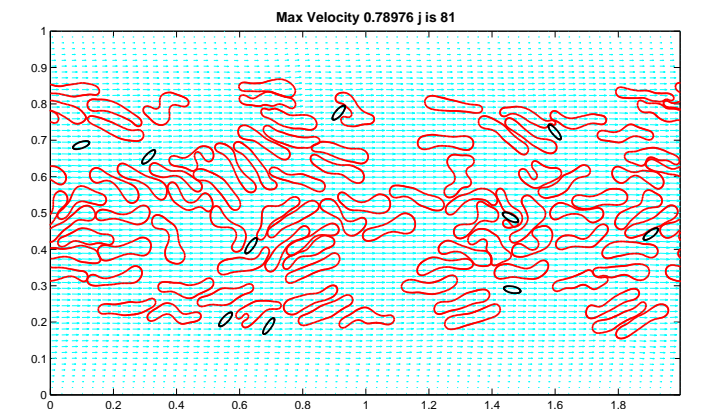
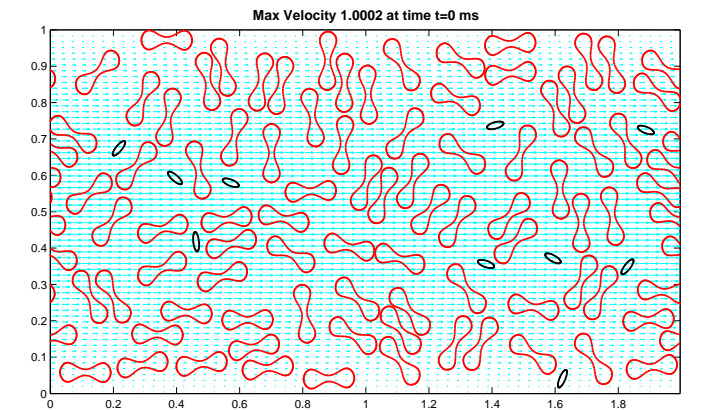
$$q = \frac{d}{dl} [E_B(\kappa(l) - \kappa_0)],$$

where $\kappa(l)$ is the instantaneous curvature and κ_0 is the curvature of minimum energy. We include the q term in order to get a biconcave red blood cell shape at rest.



Preliminary Results

This method has been implemented in Fortran 90 and parallelized using OpenMP.



Future Directions

We plan to run simulations under a wider variety of physiological conditions. IB clot obstacle.

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