Name: ______________________________

Math 1030-06
Spring 2007

Diagnostic Test
Math 1010 is a prerequisite for Math 1030. This means that you should have a working knowledge of intermediate algebra. This diagnostic test covers some of this background material. You should give yourself this test and then check your answers with the solution sheet handed out in class on Thursday.

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1. Three kinds of apples are all mixed up in a basket. How many apples must you draw (without looking) from the basket to be sure of getting at least two apples of the same kind?

   Answer: Four since the worst case senario is that you choose three apples, each of a different kind, from the barrel originally. Therefore the next one you choose must be the same type as one of the three you have already picked out.

2. There are 150 people in a class. If 80% of them are registered, how many are not registered?

   Answer: If 80% are registered, then (100-80)%=20% are not registered. Now we solve for what 20% of 150 is by the calculation:

   $$150 \times 0.2 = 30.$$

   The answer is that there are 30 people in the class who are not registered.

3. Express "three-fifths" as a fraction, a decimal, and as a percentage.

   Answer:
   $$\tfrac{3}{5} \quad = \quad 0.6 \quad = \quad 60\%.$$

4. Evaluate each of the following with $a = 4$, $b = \frac{2}{5}$, and $c = -6$:

   (a) $a \times (b + c) = 4 \times \left(\frac{2}{5} - 6\right) = -22.4.$

   (b) $a \times b + c \times \frac{a}{b} - c = 4 \times \frac{2}{5} + (-6) \times \frac{4}{2/5} - (-6) = \frac{8}{5} - 60 + 6 = -52.4.$

   (c) $5b - 3c^2 = 5 \times \frac{2}{5} - 3 \times (-6)^2 = 2 - 3 \times 36 = -106.$

5. Evaluate the following expressions on your calculator:

   (a) $(250/(34+56)) \times 27 = 75$.

   (b) $23 \times \frac{5}{7} + 6.3 \times (4^5) = 6,467.63$.

   (c) $3\sqrt{32} - \sqrt{15} = 13.0976$.

6. Simplify:

   (a) $\frac{x^5 x^2}{x^{-3}} = x^{5+2-3} = x^{10}$.

   (b) $(x^{-2}y^3)^2 = x^{-2\times2}y^{3\times2} = x^{-4}y^6$.

   (c) $(x^{-5}y^4)^2(x^0y^{-2})^2 = (x^{-10}y^8)(x^0y^{-4}) = x^{-10}y^4$.

7. If there are 0.82 US dollars in one Canadian dollar, which is smaller: one US dollar or one Canadian dollar, eh?

   Answer: The Canadian dollar is equal to \$0.82, which is less than one US dollar. Therefore the Canadian dollar is smaller.

8. One number is six times a second number. Find the numbers if their difference is 102.

   Answer: Label the first number x, and the second number y. From here we can form two equations:

   $$x = 6y \qquad \text{and} \qquad x - y = 102.$$

   Since we have two equations and two unknowns, we can solve for both. Substituting $x = 6y$ into the second equation, we have

   $$(6y) - y = 5y = 102 \text{ which implies that } y = \frac{102}{5} = 20.4.$$

   We can now solve for x using the first equation,

   $$x = 6y = 6 \times 20.4 = 122.4$$

9. If you drive at an average speed of 65 miles per hour, how long will it take you to drive 530 miles? If you can bike a distance of 45 miles in three hours and 15 minutes, what is your average biking speed in miles per hour?

   Answer: The key here is to look at the units. If we divide "miles" by "miles per hour," we get miles/(miles/hour)=hours, which is the units we want the answer to be in. If this is confusing, it may be easier to use the definition of speed to solve for time. We know that

   $$\text{Speed} = \frac{\text{Distance}}{\text{Time}},$$

and rearranging this equation by multiplying both sides by time and then dividing both sides by speed, we get the relation

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

Therefore

$$\frac{530 \text{ miles}}{65 \text{ miles per hr}} = 8.154 \text{ hours}.$$

So the answer to the first part of the question is that it will take you 8.154 hours to drive 530 miles. As for the second part of the question we are given the distance and the time and asked to solve for the speed. First we must write "three hours and 15 minutes" in terms of hours. Since 15 minutes is a fourth of an hour ($\frac{15}{60} = 0.25$), the total amount of time is 3.25 hours. Next, we can use the definition of speed to solve for this problem,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45 \text{ miles}}{3.25 \text{ hours}} = 13.85 \text{ mph}.$$

10. The length of a rectangle is 14 inches more than its width. If the area is 72 square inches, find the length and width of the rectangle.

    Answer: Let l be the length and w be the width of the rectangle. From the information given in this problem, we can create two equations. Recall that the equation for area of a rectangle is

    $$l \times w = A = 72.$$

    We also know that the difference between the length and the width of this rectangle is

    $$l - w = 14 \text{ inches}.$$

    Therefore we can solve for the system by substituting one equation into the other. First we rearrang the second equation in order to isolate l onto one side of the equation,

    $$l = 14 + w.$$

    Plugging this into the first equation, we get

    $$(14 + w) \times w = 72 \text{ or } 14w + w^2 = 72.$$

    This is a quadratic equation– we can use the quadratic formula to solve it or alternatively we can use the tool of completing the square. Using the quadratic formula,

    $$w = \frac{-14 \pm \sqrt{14^2 - 4 \times (-72)}}{2} = \frac{-14 \pm \sqrt{484}}{2} = -7 \pm 11 = 4 \text{ or } -18.$$

Using the method of completing the square (if you don't remember the quadratic formula), we want to solve for the roots a and b,

$$14w + w^2 - 72 = (w + a)(w + b) = 0,$$

$$14w + w^2 - 72 = w^2 + a \times w + b \times w + a \times b = 0$$

$$14w + w^2 - 72 = (a + b) \times w + w^2 a \times b = 0.$$

Now we need to pick integer values for a and b (by trial and error) such that $a \times b = -72$ and $a + b = 14$. The only two numbers that work turn out to be 4 and -18. Since 4 is the only positive number, the width of the rectangle must be 4. The last step is solving for the length, $l = 14 + w = 18$.

11. Suppose that three-quarters of the freshmen live in a dorm. If two-thirds of the freshmen dorm residents are women, what percentage of the freshman class are women who live in the dorm?

    Answer: Three-quarters of two-thirds is $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$. Therefore, one half are women that live in dorms. Since the question asks for the answer in terms of a percent though, we must convert the fraction to a decimal and then to a percentage. The answer is 50%.

12. Solve for x in the following equations:

    (a) $3x - 5 = 9 + 7x$: Add +5 and then subtract $7x$ from both sides. Doing these two steps we get

    $$3x - 7x = 9 + 5. \text{ Simplifying this we get } -4x = 14, \text{ i.e. } x = -\frac{14}{4} = -\frac{7}{2}.$$

    (b) $x^2 - 5 = 31$: Add +5 to both sides and then take the square root of both sides,

    $$x^2 = 31 + 5 = 36 \text{ i.e. } x = \sqrt{36} = \pm 6.$$

    (c) $x^2 - x - 12 = 0$: Either use the quadratic formula or complete the square,

    $$(x + a)(x + b) = x^2 + (a + b)x + ab = 0.$$

    We need to pick integers a and b such that $a \times b = -12$ and $a + b = -1$. This works for $a = -4$, $b = 3$. Therefore $x = 3, -4$.

    (d) $\frac{x-3}{5} = \frac{x}{2}$: Multiply both sides by $5 \times 2 = 10$ in order to get rid of the fractions,

    $$2(x - 3) = 2x - 6 = 5x.$$

    Now subtract $2x$ from both sides,

    $$-6 = 3x \Rightarrow x = -\frac{6}{3} = -2.$$

(e) $|x+3| = 10$: Since this is an absolute value problem, we can separate this into two different problems,

$$x + 3 = 10 \text{ and } x + 3 = -10.$$

Now subtract 3 from both sides of both equations,

$$x = 10 - 3 = 7 \text{ and } x = -10 - 3 = -13.$$

So the two solutions to this problem are $x = 7$ and $x = -13$.

13. Solve for x and y such that x and y satisfy the following equations: $3x - 2y = 5$ and $x + y = 7$. First we rearrange the second equation so that x is isolated (to do this we subtact y from both sides),

$$x = 7 - y.$$

Substituting this into the first equation,

$$3x - 2y = 3(7 - y) - 2y = 21 - 3y - 2y = 21 - 5y = 5.$$

Subtracting 21 from both sides we get

$$-5y = 5 - 21 = -16 \text{ i.e. } y = \frac{-16}{-5} = \frac{16}{5}.$$

Last, using the second equation again, we can now solve for x since we have a value for y,

$$x = 7 - y = 7 - \frac{16}{5} = \frac{19}{5}.$$

14. Graph the line $5x - 2y = 6$. What is the y-intercept?

Answer: The y-intercept is -3. When $x = 0$ the equation is

$$5 \times 0 - 2y = -2y = 6,$$

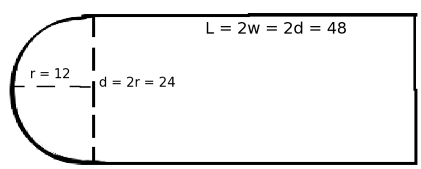
which implies that $y = \frac{6}{-2} = -3$.

15. A warehouse may contain bicycles, tricycles, and cars. Altogether there are 18 wheels in the warehouse. How many bicycles, tricycles, and cars are there? Give as many answers as possible.

Answer: The number of cars, etc. can only be a positive (or zero) integer. There are 4 wheels on a car, 2 on a bike, and 3 on a tricycle.

Number of Cars	Number of Bikes	Number of Tricycles
4	1	0
3	3	0
3	0	2
2	5	0
2	2	2
1	7	0
1	4	2
1	1	4
0	9	0
0	6	2
0	3	4
0	0	6

16. The playground drawn below is in the shape of a rectangle with a semi-circle attached as shown. Suppose that the longer side of the rectangle is twice the length of the shorter side and that the radius of the semi-circle is 12 feet. What is the perimeter and the area of the playground?



Answer: If the radius of the semicircle is 12 feet, then its diameter is 24 feet. Looking at the picture, this gives you the information that the shorter side of the rectangle (it's width) is $W = 24$ feet, and the longer side (it's length) is twice that– $L = 48$ feet. In order to find the perimeter of the whole playground, we need to know the circumference of the circle. The formula for the circumference of a circle is

$$C = 2\pi r = \pi d,$$

where r is the radius, and d is the diameter. Since we only need half of the circle's circumference is the border of the playground, we only add half of C to the perimeter of the rest of the playground.

$$\text{Perimeter of Playground} = 2 \times L + 1 \times W + \frac{C}{2}.$$

The perimeter of the playground is

$$\text{Perimeter of Playground} = 2 \times 48 + 24 + \pi \times 12 = (120 + 12\pi)\text{feet}.$$

As for the area of the playground, we can compute it in pieces. We need to find the area of the rectangle in the park and add it to the area of the semi-circle. The area of a rectangle is

$$A = L \times W = 48 \times 24 = 1152 \text{ feet}.$$

The formula for the area of a circle is

$$A = \pi r^2 = 12^2 \times \pi = 144\pi,$$

However, since we only need the area of the semi-circle (which is half of that of the circle) we just divide that number by 2. Adding the area of the rectangle to that of the semi-circle, we get

$$1152 + \frac{144}{2}\pi = (1152 + 72\pi)\text{feet}^2.$$

17. Suppose that the ratio of undergraduate students to graduate students in an institution is 18:7. What percentage of the student body are graduate students?

    Answer: If the ratio of undergraduate students to graduate students is 18 to 7, then for every 7 graduate students there are $18 + 7 = 25$ students in the student body. Therefore the fraction is

    $$\frac{7}{25} = 0.28 \text{ or } 28\%.$$

18. Suppose that your annual tuition as a freshman was \$1,856. Each year tuition has increased by 5%. Now you are in your senior year. What is your annual tuition this year?

    Answer: Every year, we add on 5% of the total amount of tuition you pay that year. By your senior year, your tuition has increased a total of 3 times.

    $$\$1,856 \times (1 + 0.05)^3 = \$2,148.55$$

19. The company you work for was doing poorly two years ago and as a result everyone took a 10% pay cut for the last year. The company is doing better now and the CEO is just promised to raise everyones salary 10% for the next year. Does this mean that your salary next year will be the same as it was two years ago? Explain.

    Answer: No! You're salary will be less than it was two years ago. Suppose that two years ago you were making D dollars a year. After the pay cut, you make 90% of D dollars. When you get the raise after the pay cut, you get 110% of 90% of D dollars, or

    $$1.10 \times (0.90 \times D) = 0.99 \times D < D.$$

    In the end you are making 99% of what you used to make two years ago.

20. Determine any errors made in the work shown below. Then explain the mistake made. The corrections have been made and are in bold. Compare them to the ones with mistakes in the original test to see the difference.

(a)

$$\begin{aligned} \frac{3(-5)+x(3)}{3} &= 1 && \Rightarrow \text{“cancel 3”} \\ -5+\mathbf{x} &= 1 && \Rightarrow \text{“add +5 to both sides”} \\ x &= 6. \end{aligned}$$

In the original problem, the first step is wrong. The 2 must first be distributed to both the x and the 3 because of the parentheses.

(b)

$$\begin{aligned} 2\left(\frac{x+3}{5}\right) &= x && \Rightarrow \text{“distribute the 2”} \\ \frac{2x+6}{5} &= x && \Rightarrow \text{“multiply by 5”} \\ 2x+6 &= 5x && \Rightarrow \text{“subtract } 2x\text{”} \\ 6 &= 5x-2x && \Rightarrow \\ 6 &= 3x && \Rightarrow \text{“divide by 3”} \\ 2 &= x \end{aligned}$$

The first step in the original problem is incorrect. You should not add -3 to both sides because the +3 is in the top part of the fraction, and adding -3 will not get rid of it. The step between lines 3 and 4 is also incorrect in the original problem. $2x$ means two times x, so subtracting 2 does not get rid of the two. In order to get rid fot he 2, you must divide by it.

(c)

$$5(x^2y^3) = 5x^2 5y^3 = 25x^2y^3.$$

This last one should be

$$5(x^2y^3) = 5x^2y^3.$$

This is because you only need to distribute the 5 if the numbers in the paratheses are being added or subtracted.