MATH 2270

Quiz #7 - Fall 2008

Name: Answer Key

1. (4 points)

(a) Find the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}.$$

$f_A(\lambda) =$

char poly: $\det(A - \lambda I_2) = \det\left(\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = \det\begin{pmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{pmatrix}$

$= (1-\lambda)(4-\lambda) + 2 = 4 - 5\lambda + \lambda^2 + 2$

$= \boxed{\lambda^2 - 5\lambda + 6}$

Factor

$f_A(\lambda) = (\lambda-2)(\lambda-3)$

(b) Determine the eigenvalues of A.

$f_A(\lambda) = 0 \iff \lambda = 2$ or $\lambda = 3$. Therefore 2 and 3 are the eigenvalues of A.

2. (3 points) Let

$$A = \begin{pmatrix} a & k \\ -1 & a \end{pmatrix}.$$

For which values of k does the matrix A have no (real) eigenvalues?

$\det \begin{pmatrix} a-\lambda & k \\ -1 & a-\lambda \end{pmatrix} = (a-\lambda)^2 + k = 0 \iff$

$\underbrace{(a-\lambda)^2}_{\text{always positive}} = -k$

$\therefore$ we need $-k \geq 0$ or $k \leq 0$ to have real eigenvalues

So, for $k > 0$ there are no real eigenvalues

3. (4 points) True or false. Indicate whether the following statements are true or false.

(a) There exists a 3×3 matrix A without any real eigenvalues.

False: The char. poly. will always have at least one real root

(b) A square matrix A is invertible if and only if 0 is *not* an eigenvalue of A.

True: 0 is not an eigenvalue if and only if $\ker(A) = \{0\}$ if and only if A is invertible.