MATH 2270
Quiz #1 - Fall 2008

Name: Answer

1. (5 points) Consider the following linear system

$$\begin{aligned} x - 2y &= 3 \\ 2x - y &= 9. \end{aligned}$$

(a) Write the corresponding augmented matrix.

$$\left(\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & -1 & 9 \end{array}\right)$$

(b) Use Gauss-Jordan elimination to convert the augmented matrix to reduced row-echelon form. Clearly show each step.

$$\left(\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & -1 & 9 \end{array}\right) -2(\mathrm{I}) \longrightarrow \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 3 & 3 \end{array}\right) \div 3 \longrightarrow$$

$$\left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 1 & 1 \end{array}\right) +2(\mathrm{II}) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \end{array}\right)$$

(c) Solve for x and y.

$$\boxed{\begin{array}{l} x = 5 \\ y = 1 \end{array}}$$

2. (4 points) True or false. Determine if the following statements are true or false.

(a) There exists a system of three linear equations with three unknowns that has exactly three solutions.

False: A linear system can have only 0, 1, or infinitely many solutions

(b) If A is a 3 x 4 matrix and $\vec{v}$ is a vector in $\mathbb{R}^4$, then the vector $A\vec{v}$ is in $\mathbb{R}^3$.

True:

$$\underset{3\times 4}{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}} \underset{4\times 1}{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}} = \underset{3\times 1}{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}}$$

3. (2 points) Let A be a 3 x 2 matrix of rank 2. Then rref(A) = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$