

§ 1.2 #31

①

$$f(t) = a + bt + ct^2 + dt^3 + et^4$$

Substituting:

$$f(1) = 1 \quad f(-1) = 5$$

$$f(2) = -1 \quad f(-2) = -29$$

$$f(3) = -59$$

gives the system:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 & -1 \\ 1 & 3 & 9 & 27 & 81 & -59 \\ 1 & -1 & 1 & -1 & 1 & 5 \\ 1 & -2 & 4 & -8 & 16 & -29 \end{pmatrix} \begin{matrix} - (I) \\ - (I) \\ - (I) \\ - (I) \end{matrix} \rightarrow$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 & 15 & -2 \\ 0 & 2 & 8 & 26 & 80 & -60 \\ 0 & -2 & 0 & -2 & 0 & 4 \\ 0 & -3 & 3 & -9 & 15 & -30 \end{pmatrix} \begin{matrix} - (I) \\ -2(II) \\ +2(IV) \\ +3(II) \end{matrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & -2 & -6 & -14 & 3 \\ 0 & 1 & 3 & 7 & 15 & -2 \\ 0 & 0 & 2 & 12 & 50 & -56 \\ 0 & 0 & 6 & 12 & 30 & 0 \\ 0 & 0 & 12 & 12 & 60 & -36 \end{pmatrix} \begin{matrix} \% 2 \end{matrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & -2 & -6 & -14 & 3 \\ 0 & 1 & 3 & 7 & 15 & -2 \\ 0 & 0 & 1 & 6 & 25 & -28 \\ 0 & 0 & 6 & 12 & 30 & 0 \\ 0 & 0 & 12 & 12 & 60 & -36 \end{pmatrix} \begin{matrix} +2(III) \\ -3(III) \\ -6(III) \\ -12(III) \end{matrix} \rightarrow$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 6 & 36 & -53 \\ 0 & 1 & 0 & -11 & -60 & 82 \\ 0 & 0 & 1 & 6 & 25 & -28 \\ 0 & 0 & 0 & -24 & -120 & 168 \\ 0 & 0 & 0 & -60 & -240 & 300 \end{array} \right) \xrightarrow{\%(-24)}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 6 & 36 & -53 \\ 0 & 1 & 0 & -11 & -60 & 82 \\ 0 & 0 & 1 & 6 & 25 & -28 \\ 0 & 0 & 0 & 1 & 5 & -7 \\ 6 & 0 & 0 & -60 & -240 & 300 \end{array} \right) \xrightarrow{\begin{array}{l} -6(\text{IV}) \\ +11(\text{IV}) \\ -6(\text{IV}) \\ +60(\text{IV}) \end{array}}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 6 & -11 \\ 0 & 1 & 0 & 0 & -5 & 5 \\ 0 & 0 & 1 & 0 & -5 & 14 \\ 0 & 0 & 0 & 1 & 5 & -7 \\ 0 & 0 & 0 & 0 & 60 & -120 \end{array} \right) \xrightarrow{\% (60)}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 6 & -11 \\ 0 & 1 & 0 & 0 & -5 & 5 \\ 0 & 0 & 1 & 0 & -5 & 14 \\ 0 & 0 & 0 & 1 & 5 & -7 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{array} \right) \xrightarrow{\begin{array}{l} -6(\text{V}) \\ +5(\text{V}) \\ +5(\text{V}) \\ -5(\text{V}) \end{array}}$$

$a = 1$
 $b = -5$
 $c = 4$
 $d = 3$
 $e = -2$

\therefore unique solution:

so that

$$\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{array} \right)$$

$$f(t) = -2t^4 + 3t^3 + 4t^2 - 5t + 1$$