

These are some review questions for the Final Exam. If you can successfully complete these TYPES of questions, you should be well prepared for the exam. I also suggest you thoroughly read the review guide for information about what material I have specifically excluded or included.

Exam 1 material

Appendix A

1. Simplify and rewrite each expression with only positive exponents

(a) $\left(\frac{2x^{-4}y^3}{y^2z^{-4}}\right)^2$

(b) $\frac{x^{-1/2}y^{2/3}}{(xy)^{2/3}}$

2. Solve the following equations

(a) $2x + x^2 - 8 = 0$

(b) $x^2 + 64 = 0$

(c) $x^2 - 81 = 0$

Chapter 1

3. Consider $(5, -10)$ and $(-1, 8)$
- (a) Find the distance between the points.
- (b) Find the midpoint of the line segment joining the points.
4. Use algebraic tests to check $x = y^2 - 8$ for symmetry with respect to both axes and the origin. Then sketch the graph of the equation.
5. Find the center and the radius of the circle $(x + 3)^2 + (y - 5)^2 = 36$.
6. Write the standard form of the equation of the circle with center $(-2, 0)$ and radius 4.
7. Find the slope of the line through the points $(2, -8)$ and $(0, 9)$.
8. Write an equation of the line that passes through $(6, -2)$ with slope $m = -\frac{1}{3}$. Write your answer in both point-slope form and slope-intercept form.
9. Write the equation of the line (in slope-intercept form) that passes through the point $(-1, 2)$ and is
- (a) parallel to $2x - 3y = 9$
- (b) perpendicular to $2x - 3y = 9$
10. Evaluate $f(x) = x^2 + 3x + 2$ at

- (a) $f(2)$
- (b) $f(t^2)$
- (c) $f(t + 1)$

11. If $f(x) = \begin{cases} 125 & \text{if } x < 0 \\ 8x + 2 & \text{if } 0 \leq x \leq 1 \\ 6 - x^2 & \text{if } 1 < x \end{cases}$ find:

- (a) $f(-20)$
- (b) $f(1)$
- (c) $f(6)$

12. Find the domain of

- (a) $f(x) = \sqrt{8x - 15}$
- (b) $h(x) = \frac{3x+13}{x^2-3x+18}$

13. Determine whether the function $f(x) = -3x\sqrt{x^2 + 10}$ is even, odd, or neither.

14. Graph the following:

- (a) $h(x) = x^3 + 5$
- (b) $f(x) = \sqrt{x - 4}$
- (c) $f(x) = \begin{cases} 3x + 7 & \text{if } x \leq -3 \\ 4x^2 - 1 & \text{if } x > -3 \end{cases}$

15. Consider $h(x)$ below. For each h , identify the parent function f and describe the sequence of transformations from f to h . Then graph h .

- (a) $h(x) = -\sqrt{x + 1} + 9$
- (b) $h(x) = -(x + 3)^2 + 1$

16. For $f(x) = 3x^2 - 7$ and $g(x) = -x^2 - 4x + 5$ calculate the following:

- (a) $(f + g)(x)$
- (b) $(f - g)(x)$
- (c) $(fg)(x)$
- (d) $\left(\frac{g}{f}\right)(x)$
- (e) $(f \circ g)(x)$
- (f) $(g \circ f)(x)$

17. Repeat the previous question for $f(x) = \frac{1}{x}$ and $g(x) = 2\sqrt{x}$.

18. Find the inverse of $f(x) = \frac{2}{2x-3}$

Exam 2 material

Chapter 2

1. Consider the function $y = -x^2 + 4x + 5$
 - (a) Does the parabola open up or down? Is the vertex a min or max?
 - (b) Find the coordinates of the vertex. Write the answer as an ordered pair.
 - (c) Find the y-intercept. Write the answer as an ordered pair.
 - (d) Find the x-intercepts, if any exist. Write the answer(s) as an ordered pair(s).
 - (e) Using the above answers graph the function.
2. Find all the real zeros of the polynomial $f(x) = 2x^3 + 9x^2 + 11x$. Determine the multiplicity of each zero and the number of turning points of the graph of the function.
3. Use long division to divide $\frac{4x^3 - 8x^2 + x + 3}{2x - 3}$
4. Verify that $(x - 4)$ and $(x + 2)$ are factors of $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$ and find the remaining factors of $f(x)$. Write the complete factorization of $f(x)$ and list all zeros.
5. Perform the operations and write the result in standard form.
 - (a) $(20 - 2i) - (6 + 4i)$
 - (b) $(10 - 3i)(4 + 12i)$
6. Write the quotient $\frac{-5+3i}{1+7i}$ in standard form.
7. List all possible rational zeros of $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$.
8. Find a polynomial function with real coefficients that has zeros: 1, -4 , $2 + 3i$.
9. Use the zero $5 - 2i$ to find all the zeros of $f(x) = x^3 - 7x^2 - x + 87$.
10. Use the rational zero test to find all zeros of the function $f(x) = x^3 + 9x^2 + 27x + 35$.
11. Consider $h(x) = \frac{x^2}{3x+1}$
 - (a) State the domain of the function
 - (b) Identify all intercepts
 - (c) Find any vertical and horizontal or slant asymptotes
 - (d) Plot some additional solution points as needed
 - (e) Sketch the graph of $f(x)$
12. Repeat the above question for $f(x) = \frac{x^2 - 2x - 8}{x^2 - 9}$

Chapter 3

13. Use the One-to-One property to solve
- (a) $e^{4x} = e^{x^2+3}$
 - (b) $\ln(6x - 13) = \ln 34$
14. (a) Write $5^{-2} = \frac{1}{25}$ in logarithmic form.
(b) Write $\log_4 4096 = 6$ in exponential form.
15. Use properties of logarithms to expand $\ln\left(\frac{\sqrt{x}y^4}{z^4}\right)$ as multiple logarithms.
16. Use properties of logarithms to condense $4[\ln z + \ln(z + 5)] - 2\ln(z - 5)$ into a single logarithm.
17. Solve the following equations algebraically.
- (a) $\log x - \log(8 - 5x) = 2$ (You must consider the domain)
 - (b) $8(3^{6-x}) + 3 = 43$
18. Use the exponential decay model $y = ae^{-bt}$ (see page 260) to answer the following question about radioactive isotopes. If we initially have 10 grams of ^{226}Ra isotope with a half-life of 1599 years, how much is left after 250 years? (Hint: you must find the initial model equation first).

Exam 3 material

Chapter 7

1. Use substitution to solve $\begin{cases} x - y = 10 \\ x - y^2 = 10 \end{cases}$
2. Use elimination (without matrices) to solve $\begin{cases} 4x + 2y = 12 \\ 5x - 3y = -5 \end{cases}$
3. Use Gaussian elimination (without matrices) to solve $\begin{cases} x - 2y + z = -6 \\ 2x - 3y = -7 \\ -x + 3y - 3z = 11 \end{cases}$
4. Solve the following system of equations using Gaussian Elimination AND Gauss-Jordan Elimination (both with matrices). $\begin{cases} -3x + y + 7z = -20 \\ 5x - 2y - z = 34 \\ -x + y + 4z = -8 \end{cases}$

5. Use the following matrices to perform the indicated matrix operations, if possible. Show your work or explain why the operation cannot be performed.

$$A = \begin{bmatrix} -4 & 3 & 8 \\ 1 & 0 & -3 \\ 6 & 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 4 \\ 1 & -3 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 4 \\ 0 & 3 \\ 6 & -2 \\ -3 & 7 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 & 8 \\ -1 & 3 & 10 \\ -4 & 1 & 2 \end{bmatrix}$$

- (a) $A + 4D =$
 (b) $AD =$
 (c) $AB =$
 (d) $CB =$
6. Consider $A = \begin{bmatrix} 9 & -3 \\ 4 & 7 \end{bmatrix}$
- (a) Find the inverse of A using the formula for a 2×2 matrix.
 (b) Find the inverse of A using the augmented matrix procedure.

7. Use the augmented matrix procedure to find the inverse of $A = \begin{bmatrix} -1 & 0 & 12 \\ 4 & -2 & 6 \\ -3 & 1 & -7 \end{bmatrix}$

8. Use inverse matrices to solve $\begin{cases} 3x - 4y = 12 \\ -x + 3y = 8 \end{cases}$

9. Find the determinant of $A = \begin{bmatrix} 4 & 0 & -1 \\ 8 & -1 & 0 \\ 0 & 3 & -2 \end{bmatrix}$

Chapter 9

10. Find the first 5 terms of the sequence $a_n = n(n^2 - 12)$
11. Determine whether the sequence $a_n = -\frac{3}{4}n + 8$ is arithmetic. Find the first 5 terms.
12. Find a formula for a_n for the arithmetic sequence with $a_1 = -3$ and $d = -2$.
13. Find the partial sum $\sum_{n=1}^{65} 4n$
14. Find the sum of the finite geometric sequence $\sum_{n=1}^{25} \left(\frac{1}{7}\right)^{n-1}$
15. Find the sum of the infinite geometric sequence $\sum_{n=0}^{\infty} 4\left(\frac{2}{9}\right)^n$
16. Use the Binomial Theorem to expand and simplify $(x + y)^5$