Math 1050-2 ~ Final Exam Review Guide*

***This is only a guide, for your benefit, and it in no way replaces class notes, homework, or studying**

<u>General Tips for Studying</u>:

1. Review this guide, class notes, and the text
2. Review comments on quizzes and exams and *<u>rework</u>* ALL quiz and exam problems
3. *<u>Review</u>* (assuming you have completed it) ALL homework *<u>assigned</u>* for Appendix A, Ch 1-3, 7-9
4. Complete ALL suggested problems for the final exam review given in class
5. Start studying early enough to ask questions!

<u>*Pay careful attention to what I have emphasized/de-emphasized (or completely eliminated)</u>

- Sections or specific topics not listed here will not be covered on the final exam
- If the directions ask you to use a certain method, you receive NO CREDIT for using another method.
- I've bolded and underlined formulas you need to memorize with: **<u>Memorize</u>**
- Pay careful attention to what I have noted in class as common mistakes on past exams and quizzes
- Keep an eye out for notes that say: this WILL be on the exam

<u>Chapter A: Review of Fundamental Concepts of Algebra</u>

*There will no direct questions from the Appendix since it is all review, but since you need this material complete the future material it is VITAL that you understand everything in this chapter.

- **<u>Commonly misused material (mistakes) from Appendix A</u>**:
  - Be able to simplify algebraic expressions by combining like terms and removing symbols of grouping (DISTRIBUTE your negative signs!)
  - Remember! Keep track of whether your negative signs are being raised to the power along with your real number:

    $(-1)^2 = (-1)(-1) = 1$ BUT $-1^2 = -(1)^2 = -(1)(1) = -1$
    - In the first case, the negative is being squared, but in the second case it is not
  - Should know and be comfortable using ALL of the Properties of Exponents (see handout from first day, table page A11) for integers AND fractions.
    - VERY important rules:
      - $a^m \cdot a^n = a^{m+n}$
      - $(a^m)^n = a^{mn}$
      - $(ab)^m = a^m \cdot b^m$
      - $a^{-m} = 1/a^m$
  - Be able to rewrite exponential expressions with only positive exponents and simplify (note that these problems do NOT involve radicals in any way!)
  - Know how to evaluate the nth roots of numbers and evaluate radical expressions
  - Know ALL Properties of Radicals (see the handout from the first day, table page A15)
  - Know the special product formulas from page A25 for convenience
    - $(u+v)(u-v)=u^2-v^2$ (you better believe you'll see difference of squares on the final!)
    - $(u+v)^2=(u+v)(u+v)=u^2+2uv+v^2$ AND $(u-v)^2=(u-v)(u-v)=u^2-2uv+v^2$
  - Always factor out the greatest common monomial factor (g.c.m.f) from polynomials if possible
  - Factor the difference of two squares **<u>Memorize</u>**: $u^2-v^2=(u+v)(u-v)$
  - Perfect square trinomials: You can simply factor perfect square trinomials as you would any other trinomial, but if you do memorize the formulas, you can save time.
  - Factor trinomials of the form x^2+bx+c and ax^2+bx+c
    - Remember! You want 2 factors of c whose sum is b.

      $(x+m)(x+n)= x^2+(m+n)x+mn = x^2+bx+c$
    - When $a \neq 1$: You must actually plug in your choices and check if the middle term is right.
    - Remember! Factors of a go in front of the x, factors of c go in place of m and n

      ax2+bx+c=(____x+____)(____x+____)

- Find the domain (x values we can plug in) of a rational function
    - Look for values of x where the denominator is zero and exclude these values
- Simplify rational expressions
    - Steps: 1. Completely factor the numerator and the denominator 2. Divide out any ***factors*** that are common to both the numerator and the denominator
    - **Remember!** We can only divide out expressions that are multiplying other expressions (**factors**). We CANNOT divide out ***terms***, which are expressions that are being added or subtracted to other expressions.
- Remember! If the rational expressions have unlike denominators, we MUST get a common denominator (the LCD) before we can add or subtract
- Use factoring and the zero-factor property to solve quadratic equations
    - **Remember!** The zero-factor property only holds when the RHS=0
- Solve quadratic equations by using the Square Root Property (both real and complex)
    - **Memorize**: If $u^2=d$ where $d>0$ then $u=\sqrt{d}$ and $u=-\sqrt{d}$
- Use the quadratic formula to solve quadratic equations
    - **Memorize**: The solutions of $ax^2+bx+c=0$ are given by $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Chapter 1: Functions and Their Graphs

- 1.1 Rectangular Coordinates:
    - Know how to plot points on a rectangular coordinate system
    - Be able to determine whether given ordered pairs are solutions of a given equation
    - **Memorize** and know how to use the distance formula and the midpoint formula
    - Know how to use these formulas in application questions

- 1.2 Graphs of Equations:
    - Be able to find the x- and y-intercepts of an equation, write them as ordered pairs, and then use the intercepts to plot a graph of the equation
    - Understand both the graphical and the algebraic tests for symmetry
        - x-axis symmetry: Geometric: whenever (x,y) is on the graph, so is (x,-y). Algebraic: replacing y with -y in our equation yields an equivalent equation
        - y-axis symmetry: Geometric: whenever (x,y) is on the graph, so is (-x,y). Algebraic: replacing x with -x in our equation yields an equivalent equation
        - origin symmetry: Geometric: whenever (x,y) is on the graph, so is (-x,-y). Algebraic: replacing x with -x AND y with -y in our equation yields an equivalent equation
    - **Memorize** and use the standard form of a circle to find the radius and center of the circle. Be able to use the center and radius to write the equation for a circle.

      $(x-h)^2+(y-k)^2=r^2$ where r is the radius and (h,k) is the center.

- 1.3 Linear Equations in Two Variables:
    - Know how to find the slope of a (nonvertical) line through two points
        - **Memorize**: m= $(y_2 - y_1) / (x_2 - x_1)$
        - **Remember!** You can label either point (x_1, y_1), but once you do, order matters!
        - **Remember!** Vertical lines have undefined slope, horizontal lines have 0 slope
        - **Remember!** Lines with positive slope rise from left to right, lines with negative slope fall from left to right
    - **Memorize**: Slope-Intercept form: y = mx + b (now just read off m=slope, (0,b)=y-intercept)
    - **Memorize**: Point-Slope form: $y - y_1 = m(x - x_1)$
        - Find the equation of a line given the slope and one point on the line
        - **Remember!** Point-Slope form simplifies to Slope-Intercept form
    - Use slope to determine whether lines are parallel or perpendicular

- **Remember!** Parallel lines have the same slope ($m_1=m_2$) and perpendicular lines have slopes that are negative reciprocals ($m_1= -1/m_2$)

- Understand problems of the form: write the equation of the line through (x,y) that is parallel or perpendicular to a given line

- 1.4 Functions:
  - Know the difference between a relation and a function
    - Remember! In order for a relation to be a function, each element of the domain is matched with <u>exactly one</u> element of the range
  - Be able to understand and use function notation and evaluate functions (including piece-wise functions!)
  - Be able to find the domain and range of functions
    - Remember! The domain is what you can plug in to a function, the range is what you get out of the function
    - You should watch out for zero denominators and negative expressions under even indexed radicals (ie: square roots, 4^{th} roots, etc.)

- 1.5 Analyzing Graphs of Functions:
  - Be able to recognize basic functions and understand the vertical line test
  - Know how to find the zeros of functions
  - Be able to look at a function with labeled intervals and determine whether the function is increasing, decreasing or constant on a particular interval
  - Be able to test whether functions are even (y-axis symmetry) or odd (origin symmetry)
    - Even: $f(-x)=f(x)$
    - Odd: $f(-x)=-f(x)$

- 1.6 A Library of Parent Functions:
  - Be familiar with the functions from this section, their properties, and know what their graphs look like (linear, squaring, cubic, square root, reciprocal, absolute value).
  - Be able to graph these parent functions and piecewise functions.

- 1.7 Transformations of Functions ($c>0$ unless noted):
  - Vertical shift c units up: $h(x)=f(x)+c$
  - Vertical shift c units down: $h(x)=f(x)-c$
  - Horizontal shift c units right: $h(x)=f(x-c)$
  - Horizontal shift c units left: $h(x)=f(x+c)$
  - Reflection in x-axis: $h(x)=-f(x)$
  - Reflection in y-axis: $h(x)=f(-x)$

- 1.8 Combinations of Functions (this **WILL** be on the final):
  - Know how to arithmetically combine functions
    - $(f+g)(x)=f(x)+g(x)$
    - $(f-g)(x)=f(x)-g(x)$
    - $(fg)(x)=f(x)\cdot g(x)$
    - $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
  - Understand function composition: $(f\circ g)(x)=f(g(x))$
    - This is read f of g of x
    - Remember! Function composition is NOT multiplication!
  - Be able to find the domains of any of these functions

- 1.9 Inverse Functions:
  - Understand the definition of an inverse function and be able to use that definition to verify that two functions are inverses of each other
    - $(f \circ g)(x) = f(g(x)) = x$ AND $(g \circ f)(x) = g(f(x)) = x$
  - Be able to apply the horizontal line test to see if a function is one-to-one
  - Know what it means for a function to be one-to-one (each input corresponds to exactly one output)
    - **Remember!** A function has an inverse if and only if it is one-to-one
  - Be able to find the inverse of function algebraically:
    - Determine whether the function has an inverse
    - Replace f(x) with y
    - Interchange x and y, then solve for y
    - Replace y with $f^{-1}(x)$

<u>Chapter 2: Polynomial and Rational Functions</u>

2.1 Quadratic Functions and Models:

- Be able to sketch parabolas, $f(x) = ax^2 + bx + c$, using the following information:
  - The parabola opens up if $a > 0$ (so the vertex is a minimum) and down if $a < 0$ (so the vertex is a maximum).
  - **<u>Memorize!</u>** The vertex of a parabola is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
  - Find the y-intercept of the parabola. Write as an **<u>ordered pair</u>**. Let $x=0$ and solve for y . Notice this always gives you $y=c$, so the y-intercept is $(0,c)$.
  - Find the x-intercept(s) of the parabola. Write as an **<u>ordered pair(s)</u>**. Let $y=0$ and solve for x . You can either try to factor or just use the quadratic formula (I suggest the latter).
  - There is a handout with more detail and a sample question on the website.
- Understand how to use quadratic equations in applications (limit your study of this based on the homework questions I assigned here).

- 2.2 Polynomial Functions of Higher Degree:
  - Be able to use the leading coefficient test to describe the right-hand and left-hand behavior of the graph of a polynomial function (see table page 141).
  - Know how to find a polynomial with a given degree and given zeros.
  - Understand how to find zeros of polynomials and their multiplicities. Then be able to determine the number of turning points a given graph has based on those multiplicities.
  - Be able to sketch the graphs of higher degree polynomials using the following steps:
    - apply the leading coefficient test
    - find the zeros and their multiplicities
    - plot some extra points
    - draw a continuous curve

- 2.3 Polynomial and Synthetic Division:
  - Be able to use long division to divide polynomials by polynomials
    - Steps: 1. Write the dividend and divisor in descending powers of the variable. 2. Insert placeholders with zero coefficients for missing powers of the variable. 3. Perform long division of the polynomials as you would with integers. 4. Continue the process until the degree of the remainder is less than that of the divisor.
    - Remember to write your answer as: $q(x) + \frac{r(x)}{d(x)}$
  - Use synthetic division to divide polynomials by polynomials of the form $(x-k)$
    - Remember! k can be positive OR negative

- Know how to use synthetic division to factor polynomials using the Factor Theorem:
  - A polynomial $f(x)$ has a factor $(x-k)$ iff $f(k)=0$

- 2.4 Complex Numbers:
  - Be able to write complex numbers and perform operations with these numbers
    - $i=\sqrt{-1}$ so that $i^2=-1$
    - This means that $\sqrt{-c}=\sqrt{c(-1)}=\sqrt{c}\sqrt{-1}=i\sqrt{c}$
  - Know how to add, subtract, and multiply (use FOIL) complex numbers
  - Understand what complex conjugates are and be able to use them to write the quotient of two complex numbers in standard form $a+bi$
    - Remember! $(a+bi)(a-bi)=a^2+b^2$ (pay special attention the the + sign here)
  - Make note that these skills allow us to find complex zeros of polynomials (we can still use the quadratic formula to solve quadratic equations).

- 2.5 Zeros of Polynomial Functions:
  - Remember that the Fundamental Theorem of Algebra tells us that an nth degree polynomial has exactly n roots. As a result, the polynomial can be factored into exactly n linear factors.
  - Know how to apply the rational zero test (page 170) to find all roots (real and complex) of a given polynomial, and then factor that polynomial completely.
    - You may be asked to list all possible rational zeros.
    - Remember! This involves synthetic division.

- 2.6 Rational Functions (Commonly missed topic on Exam 2 **likely** to show up on the final):
  - Know how to find the domain of rational functions
  - Be able to find the vertical, horizontal, and slant asymptotes of rational functions (see the rules on page 186).
    - Remember! If a function has a slant asymptote, it does not have a horizontal
  - Know how to analyze graphs of rational functions based on the 7 rules on page 187. This takes you step by step through how to graph a rational function.

<u>Chapter 3: Exponential and Logarithmic Functions</u>

- 3.1 Exponential Functions and Their Graphs:
  - Be able to use the One-to-One property to solve exponential equations (when applicable).

- 3.2 Logarithmic Functions and Their Graphs:
  - **<u>Memorize</u>** the very important rule: $y=\log_a x$ iff $x=a^y$
  - Be able to use the above rule to convert equations for logarithmic functions to exponential form and vice versa.
  - Know how to use the One-to-One property to solve logarithmic equations (when applicable).
    - **Remember!** This property only works for equations of the form: $\log(x)=\log(y)$ NOT one of the form $\log(x)+\log(y)=\log(z)$

- 3.3 Properties of Logarithms:
  - Be especially sure you can utilize the Properties of Logarithms (page 240) to evaluate or rewrite logarithmic expression and to expand or condense logarithmic expressions. (There **WILL** be a question testing you on these rules!)

- 3.4 Exponential and Logarithmic Equations:
  - Solve simple exponential and logarithmic equations by using the One-to-One property or the Inverse Property

- Use the following methods for solving exponential and logarithmic equations in general:
    - Rewrite an *exponential* equation in logarithmic form and apply the inverse property of logarithmic functions. (Think of this as taking the logarithm of each side of the equation).
    - Rewrite a *logarithmic* equation in exponential form and apply the inverse property of exponential functions. (Think of this as exponentiating both sides of the equation).
    - You **WILL** be required to solve an exponential and/or logarithmic equation on the final. It may be in the context of an application question (see 3.5).

- 3.5 Exponential and Logarithmic Models:
  - Recognize the 5 most common types of models (applications) that use exponential and logarithmic functions.
  - Focus on the applications that were assigned in the homework. Take particular note of: compound interest, radioactive isotopes, carbon dating, and bacterial growth.
    - Formulas will be given if needed

<u>Chapter 7: Systems of Equations</u>

- 7.1 Linear and Nonlinear Systems of Equations:
  - Determine if ordered pairs are solutions of systems of equations
    - Remember! (x,y) must solve ALL equations of the system to be a solution
  - Solve systems of 2 equations (especially lines) graphically
  - Solve systems (BOTH linear and nonlinear) of 2 equations using substitution

- 7.2 Two-Variable *Linear* Systems:
  - Solve <u>linear</u> systems of 2 equations using elimination

Remember! <u>Linear</u> systems have only 1 solution, infinitely many solutions, or no solutions

- 7.3 Multivariable *Linear* Systems:
  - Use Gaussian Elimination (without matrices) to reduce a system to row-echelon form
  - Be able to use back-substitution to solve a system after you put it in row-echelon form.

<u>Chapter 8: Matrices and Determinants</u>

- 8.1 Matrices and Systems of Equations:
  - Know how to determine the order of a matrix
  - Be able to form coefficient and augmented matrices from linear systems and vice versa
  - Know and understand how to use the elementary row operations
  - Know the difference between row-echelon form and reduced row echelon form
  - Solve systems of equations using <u>Gaussian Elimination</u> with back-substitution (with matrices)
    - 1. Write the augmented matrix 2. Use elementary row operations to rewrite the augmented matrix in row-echelon form 3. Write the system of linear equations for this matrix, and use back-substitution to find the solution.
  - Be able to solve systems of equations using <u>Gauss-Jordan Elimination</u> (with matrices)
    - 1. Write the augmented matrix 2. Use elementary row operations to rewrite the augmented matrix in <u>reduced</u> row-echelon form 3. Read off the solution
  - See the handouts from class for more details on completing these steps
  - Remember! Order matters! I suggest working top to bottom and left to right
  - **You will be asked to solve a system of 3 equations on the final exam.** You should know how to do so using 3 methods: Gaussian Elimination without matrices, Gaussian Elimination with back-substitution using matrices, and Gauss-Jordan Elimination with matrices. I may or may not specify the method to be used.

- 8.2 Operations with Matrices (this **WILL** be on the final):
  - Know what it means for two matrices to be equal
  - Be able to add, subtract, and scalar multiply matrices and to multiply two matrices
  - Be sure you know when the above operations can and cannot be performed

- 8.3 The Inverse of a *Square* Matrix:
  - Be able to use the definition of matrix inverses to verify two matrices are inverses
  - <u>Memorize</u> the formula for the inverse of a 2x2 matrix
  - Know how to use Gauss-Jordan Elimination to find the inverses of 2x2 and larger matrices. Also be able to describe in words how you would find the inverse of a large (say, 100x100) matrix.
    - 1. Form the augmented matrix [A|I], where A is the $n \times n$ matrix and I is the $n \times n$ identity matrix. 2. Perform elementary row operations on [A|I] until we have an augmented matrix of the form [I|B]. That is, until the matrix A on the left is transformed into the identity matrix. 3. The matrix B (on the right) is the inverse of A.
  - Be able to use inverse matrices to solve 2 variable systems of linear equations

- 8.4 The Determinant of a *Square* Matrix:
  - Know how to find the determinant of 2x2 and 3x3 matrices

<u>Chapter 9: Sequences and Series</u> (There will be only 1 or 2 questions on the final from this chapter)
See the handout from class for these formulas do NOT memorize!

- 9.1 Sequences and Series:
  - Be able to list several terms of a sequence (even those written in summation notation)
  - Know how to simplify factorial expressions

- 9.2 Arithmetic Sequences and Partial Sums:
  - Know how to recognize and write arithmetic sequences
  - Be able to find the n^{th} term and the n^{th} partial sum of an arithmetic sequence

- 9.3 Geometric Sequences and Series:
  - Know how to recognize and write geometric sequences
  - Be able to find the n^{th} term and the n^{th} partial sum of a geometric sequence
  - Be able to find the sum of an infinite geometric series

- 9.5 The Binomial Theorem:
  - Be able to expand binomials using the Binomial Theorem