Math 1050-2 ~ Exam #1 Review Guide*

***This is only a guide, for your benefit, and it in no way replaces class notes, homework, or studying**

<u>General Tips for Studying</u>:

1. Review this guide, class notes, the text, and examples done in class and in the text
2. Review comments on quizzes and *<u>rework</u>* ALL quiz problems
3. *<u>Review</u>* (assuming you have completed it) ALL homework assigned for Appendix A and Ch 1
4. Complete ALL suggested problems for the exam review given in class
5. Start studying early enough to ask questions!

<u>Helpful Definitions</u>:

Rework: write down the problem and solve it yourself, then check your work

Review: look over the material making note of what you comprehend, rework what you do not

<u>Chapter A: Review of Fundamental Concepts of Algebra</u>

*There will probably be few direct questions from the Appendix since it is all review, but since you need this material complete the future material it is VITAL that you understand the following:

- A.1 Real Numbers and Their Properties:
  - Understand the set of real numbers
  - Be able to use the real number line to order real numbers
  - Determine the absolute value of a real number
    - $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a \leq 0 \end{cases}$
    - Remember! Absolute value is the <u>distance</u> from a to 0, so it is <u>always</u> positive!
  - Know how to find the distance between two real numbers: distance = $|a-b|=|b-a|$
  - Understand terms (constant and variable) and coefficients of algebraic expressions
    - Remember! Algebraic expressions do not contain equals signs
  - Be able to simplify algebraic expressions by combining like terms and removing symbols of grouping
  - Know how to evaluate algebraic expressions (substitute values in for variables)
  - Be able to add, subtract, multiply, and divide real numbers
    - Sometimes it helps to think of subtraction as addition:

      $a-b=a+(-b)$
    - Remember! When adding and subtracting fractions, we <u>must</u> have a common denominator first. Know how to find the least common denominator (LCD)
    - Division is just multiplication by the reciprocal:

      $a \div b = a * (1/b) \qquad (b \neq 0)$
  - You should understand and know how to use all of the properties discussed in this section (see the handout from the first day of class and tables pages A6, A7)
  - I will NOT test you on specific names or have you identify the properties, but you must know what you can and cannot do when you are working with real numbers (this is especially true when manipulating algebraic expressions and equations).

- A.2 Exponents and Radicals:
  - Know how to write repeated multiplication in exponential form and evaluate
    - $a^n = a * a * a \cdots a \qquad$ (n times)
    - Remember! Keep track of whether your negative signs are being raised to the

power along with your real number:

$(-1)^2 = (-1)(-1) = 1$ BUT $-1^2 = -(1)^2 = -(1)(1) = -1$

- In the first case, the negative is being squared, but in the second case it is not

- Should know and be comfortable using ALL of the Properties of Exponents (see handout from first day, table page A11) for integers AND fractions.
  - VERY important rules:
    - $a^m \cdot a^n = a^{m+n}$
    - $(a^m)^n = a^{mn}$
    - $(ab)^m = a^m \cdot b^m$
    - $a^{-m} = 1/a^m$
- Be able to rewrite exponential expressions with only positive exponents and simplify (note that these problems do NOT involve radicals in any way!)
- Know how to evaluate the nth roots of numbers and evaluate radical expressions
- Know ALL Properties of Radicals (see the handout from the first day, table page A15)
- Understand how to change exponents to radicals and radicals to exponents
  - $a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$ AND $a^{m/n} = (a^m)^{1/n} = (\sqrt[n]{a^m})$
- Be able to find the domain of a radical function
  - If n is odd, the domain of $f(x) = \sqrt[n]{x}$ is all real numbers
  - If n is even, the domain of $f(x) = \sqrt[n]{x}$ is all nonnegative real numbers
- Be able to rationalize the denominators (to remove any radicals) of radical expressions

- A.3 Polynomials and Factoring:
  - Identify leading coefficient, degree, constant term, and standard form of a polynomial
  - Add and subtract polynomials,
  - Use the distributive property and FOIL to multiply polynomials
  - Know the special product formulas from page A25 for convenience (you don't have to memorize these, you can just multiply them out if you wish)
    - $(u+v)(u-v) = u^2 - v^2$
    - $(u+v)^2 = (u+v)(u+v) = u^2 + 2uv + v^2$
    - $(u-v)^2 = (u-v)(u-v) = u^2 - 2uv + v^2$
  - Factor out the greatest common monomial factor (g.c.m.f) from polynomials
  - Factor polynomials by grouping
    - Use this for polynomials with 4 terms only!
  - Factor the difference of two squares **Memorize**: $u^2 - v^2 = (u+v)(u-v)$
  - Factor completely by factoring repeatedly until you cannot factor the result any further
  - Perfect square trinomials: You can simply factor perfect square trinomials as you would any other trinomial, but if you do memorize the formulas, you can save time.
  - Factor trinomials of the form x^2+bx+c and ax^2+bx+c
    - Remember! You want 2 factors of c whose sum is b.

      $$(x+m)(x+n) = x^2 + (m+n)x + mn = x^2 + bx + c$$
    - Tips: 1. If c is *positive*, its factors have like signs that match the sign of b
      2. If c is *negative*, its factors have unlike signs
    - When $a \neq 1$: You must actually plug in your choices and check if the middle term is right.
    - Remember! Factors of a go in front of the x, factors of c go in place of m and n

      ax²+bx+c=(____x+____)(____x+____)

- A.4 Rational Expressions:
  - Find the domain (x values we can plug in) of a rational function
    - Look for values of x where the denominator is zero and exclude these values

- Simplify rational expressions
  - Steps: 1. Completely factor the numerator and the denominator 2. Divide out any ***factors*** that are common to both the numerator and the denominator
  - **Remember!** We can only divide out expressions that are multiplying other expressions (**factors**). We CANNOT divide out ***terms***, which are expressions that are being added or subtracted to other expressions.
- Be able to add, subtract, multiply and divide rational expressions and simplify
- Remember! If the rational expressions have unlike denominators, we MUST get a common denominator (the LCD) before we can add or subtract
- Be able to simplify complex fractions using rules for dividing rational expressions

◆ A.5 Solving Equations:
- Remember! Equations are algebraic expressions connected by an equals sign
- Remember! Solutions of an equation are values of the variable involved that satisfy the equation (meaning the equals sign holds)
- Know how to check whether a given value of a variable is a solution to an equation
- Know how to solve linear equations by maintaining the balance of the equation by forming equivalent equations using only the rules from page A47
  - Simplify either side
  - Add or subtract the same quantity to BOTH sides of the equation
  - Multiply or divide EACH side of the equation by the same nonzero quantity
  - Interchange the two sides of the equation
- Be able to solve rational equations with constant or variable denominators
- Remember! The first step is to multiply each term by the LCD of ALL fractions involved (NOT the LCD/LCD)
  - We are not trying to get a common denominator, we are trying to eliminate all denominators (so the resulting equation is either linear or quadratic)
- Remember! You MUST check for extraneous solutions. In these problems, that means we must check if our answer(s) give a zero denominator in the ORIGINAL equation. If an answer does, then it is not a solution of the equation.
- Use factoring and the zero-factor property to solve quadratic equations
  - Remember! The zero-factor property only holds when the RHS=0
- Guidelines:
  - Write the equation in general form (the RHS=0)
  - Factor the left side of the equation
  - Set each factor with a variable equal to zero (apply the zero-factor property)
  - Solve each resulting linear equation
  - Check the solution in the original equation
- Solve quadratic equations by using the Square Root Property (both real and complex)
  - If $u^2=d$ where $d>0$ then $u=\sqrt{d}$ and $u=-\sqrt{d}$
- Rewrite quadratic expressions in completed square form and be able to solve quadratic equations by completing the square
  - To complete the square for x^2+bx add $(b/2)^2$ to see $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$
- Use the quadratic formula to solve quadratic equations
  - The solutions of $ax^2+bx+c=0$ are given by $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- Be able to solve radical equations

- **For single radical equations**: isolate the radical and then raise each side of the equation to the appropriate nth power (2 for square roots, 3 for cube roots). Then, solve the resulting equation.
    - **For multiple radical equations**: try to isolate a radical on each side, then follow the steps as above. If you cannot isolate the radicals then raise each side to the nth power, but notice that there will be a radical in the resulting equation. You must then isolate this new radical and do the process again. (This type of equation will likely show up on the exam!)
  - Remember! You MUST check for extraneous solutions. In these types of problems, extraneous solutions are ones that simply do not solve the ORIGINAL equation. Raising to the nth power sometimes introduces these extra "solutions."
    - ALWAYS check the solutions in the original equation, do NOT raise both sides to the nth power while you are checking!!!
  - Be able to solve absolute value equations
    - $|x| = a \Rightarrow x = -a$ or $x = a$ (remember! $a \geq 0$)
    - $|ax+b| = |cx+d| \Rightarrow ax+b = cx+d$ and $ax+b= -(cx+d)$

- A.6 Linear Inequalities in One Variable:
  - Be able to sketch graphs of inequalities
    - Remember! [or] indicate that the endpoint is included in the interval, which corresponds to $\geq$ or $\leq$
    - Remember! (or) indicate that the endpoint is NOT included in the interval, which corresponds to > or <
  - Be able to solve linear inequalities (this is much like solving equations)
    - Remember! When you multiply or divide by a negative number you must switch the direction of your inequalities sign!
  - Be able to solve inequalities involving absolute values (don't memorize for the exam)
    - $|x| < a$ iff $-a < x < a$ (also holds for $\leq$)
    - $|x| > a$ iff $x < -a$ or $x > a$ (also holds for $\leq$ and $\geq$)

- A.7 Errors and the Algebra of Calculus:
  - Look over these common errors so you are sure to avoid them on the exam

Chapter 1: Functions and Their Graphs

- 1.1 Rectangular Coordinates:
  - Know how to plot points on a rectangular coordinate system
  - Be able to determine whether given ordered pairs are solutions of a given equation
  - Memorize and know how to use the distance formula and the midpoint formula
  - Know the Pythagorean Theorem and how to use it to determine whether a given triangle is a right triangle
  - Know how to use these formulas in application questions

- 1.2 Graphs of Equations:
  - Sketch graphs of equations using the point-plotting method (building a table of values)
  - Be able to find the x- and y-intercepts of an equation, write them as ordered pairs, and then use the intercepts to plot a graph of the equation
  - Understand both the graphical and the algebraic tests for symmetry
    - x-axis symmetry: Geometric: whenever (x,y) is on the graph, so is (x,-y).

Algebraic: replacing y with -y in our equation yields an equivalent equation
    - y-axis symmetry: Geometric: whenever (x,y) is on the graph, so is (-x,y). Algebraic: replacing x with -x in our equation yields an equivalent equation
    - origin symmetry: Geometric: whenever (x,y) is on the graph, so is (-x,-y). Algebraic: replacing x with -x AND y with -y in our equation yields an equivalent equation
- Be able to use the standard form of a circle to find the radius and center of the circle and be able to use the center and radius to write the equation for a circle.
  $(x-h)^2+(y-k)^2=r^2$ where r is the radius and (h,k) is the center.

- 1.3 Linear Equations in Two Variables:
  - Know how to find the slope of a (nonvertical) line through two points
    - $m = (y_2 - y_1) / (x_2 - x_1)$
    - Remember! You can label either point (x_1, y_1), but once you do, order matters!
    - Remember! Vertical lines have undefined slope, horizontal lines have 0 slope
    - Remember! Lines with positive slope rise from left to right, lines with negative slope fall from left to right
  - Slope-Intercept form: $y = mx + b$ (now just read off m=slope, (0,b)=y-intercept)
  - Point-Slope form: $y - y_1 = m(x - x_1)$
    - Find the equation of a line given the slope and one point on the line
    - Remember! Point-Slope form simplifies to Slope-Intercept form
  - Use slope to determine whether lines are parallel or perpendicular
    - Remember! Parallel lines have the same slope ($m_1=m_2$) and perpendicular lines have slopes that are negative reciprocals ($m_1= - 1/m_2$)
  - Understand problems of the form: write the equation of the line through (x,y) that is parallel or perpendicular to a given line

- 1.4 Functions:
  - Know the difference between a relation and a function
    - Remember! In order for a relation to be a function, each element of the domain is matched with exactly one element of the range
  - Be able to understand and use function notation and evaluate functions (including piece-wise functions!)
  - Be able to find the domain and range of functions
    - Remember! The domain is what you can plug in to a function, the range is what you get out of the function
    - You should watch out for zero denominators and negative expressions under even indexed radicals (ie: square roots, 4^{th} roots, etc.)
  - Know how to evaluate and simplify difference quotients

- 1.5 Analyzing Graphs of Functions:
  - Be able to recognize basic functions and understand the vertical line test
  - Know how to find the zeros of functions
  - Be able to look at a function with labeled intervals and determine whether the function is increasing, decreasing or constant on a particular interval
  - Know the average rate of change of f from x_1 to x_2 = $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

- Be able to test whether functions are even (y-axis symmetry) or odd (origin symmetry)
  - Even: $f(-x)=f(x)$
  - Odd: $f(-x)=-f(x)$

- 1.6 A Library of Parent Functions:
  - Be familiar with the functions from this section, their properties, and know what their graphs look like (linear, squaring, cubic, square root, reciprocal, absolute value).
  - Be able to graph these parent functions and piecewise functions.
  - Understand the greatest integer function $f(x)=[\![x]\!]$ and be able to evaluate step functions.

- 1.7 Transformations of Functions ($c>0$ unless noted):
  - Vertical shift c units up: $h(x)=f(x)+c$
  - Vertical shift c units down: $h(x)=f(x)-c$
  - Horizontal shift c units right: $h(x)=f(x-c)$
  - Horizontal shift c units left: $h(x)=f(x+c)$
  - Vertical stretch: $g(x)=cf(x) \quad c>1$
  - Vertical shrink: $g(x)=cf(x) \quad 0<c<1$
  - Horizontal stretch: $g(x)=f(cx) \quad 0<c<1$
  - Horizontal shrink: $g(x)=f(cx) \quad c>1$

  - Reflection in x-axis: $h(x)=-f(x)$
  - Reflection in y-axis: $h(x)=f(-x)$

- 1.8 Combinations of Functions:
  - Know how to arithmetically combine functions
    - $(f+g)(x)=f(x)+g(x)$
    - $(f-g)(x)=f(x)-g(x)$
    - $(fg)(x)=f(x)\cdot g(x)$
    - $(\frac{f}{g})(x)=\frac{f(x)}{g(x)}$

  - Understand function composition: $(f\circ g)(x)=f(g(x))$
    - This is read f of g of x
    - Remember! Function composition is NOT multiplication!
  - Be able to find the domains of any of these functions

- 1.9 Inverse Functions:
  - Understand the definition of an inverse function and be able to use that definition to verify that two functions are inverses of each other
    - $(f\circ g)(x)=f(g(x))=x$ AND $(g\circ f)(x)=g(f(x))=x$
  - Be able to apply the horizontal line test to see if a function is one-to-one
  - Know what it means for a function to be one-to-one (each input corresponds to exactly one output)
    - Remember! A function has an inverse if and only if it is one-to-one
  - Be able to find the inverse of function algebraically:
    - Determine whether the function has an inverse
    - Replace f(x) with y
    - Interchange x and y, then solve for y
    - Replace y with $f^{-1}(x)$