# Math 1210-9 ~ Final Exam Review Guide*

***This is only a guide, for your benefit, and it in no way replaces class notes, homework, or studying**

**General Tips for Studying:**

1. Review this guide, class notes, and the text
2. Review comments on quizzes and exams and *rework* ALL quiz and exam problems
3. *Review* (assuming you have completed it) ALL homework assigned for the sections that will covered on the exam: 0.5-0.7, N.1-N.5, 1.1, 1.3-3.6, 3.8-4.5, 5.1-5.5
4. Complete ALL suggested problems for the final exam review given in class
5. Start studying early enough to ask questions!

***Pay careful attention to what I have emphasized/de-emphasized (or completely eliminated)**

- Sections or specific topics not listed here will NOT be covered on the final exam
- If the directions ask you to use a certain method, you receive NO CREDIT for using another method.
- I've bolded and underlined formulas you need to memorize with: **Memorize**
- Pay careful attention to what I have noted in class as common mistakes on past exams and quizzes
- Keep an eye out for notes that say: this WILL be on the exam

## Chapter 0: Preliminaries

*There will not be direct questions from Chapter 0 since it is all review, but you need this material to complete the future material, so it is VITAL that you understand this entire chapter.

- 0.5 Functions and Their Graphs:
  - Be able to find the domain of a function
  - Be able to test whether functions are even (y-axis symmetry) or odd (origin symmetry)
    - Even: $f(-x)=f(x)$
    - Odd: $f(-x)=-f(x)$

- 0.6 Operations on Functions:
  - Be familiar with the basic parent functions, and know what their graphs look like (linear, squaring, cubic, square root, reciprocal, absolute value).
  - Be able to graph piecewise functions.
  - Useful shifts:
  - Vertical shift c units up: $h(x)=f(x)+c$
  - Vertical shift c units down: $h(x)=f(x)-c$
  - Horizontal shift c units right: $h(x)=f(x-c)$
  - Horizontal shift c units left: $h(x)=f(x+c)$
  - Reflection in x-axis: $h(x)=-f(x)$
  - Reflection in y-axis: $h(x)=f(-x)$
  - Know how to arithmetically combine functions
    - $(f+g)(x)=f(x)+g(x)$
    - $(f-g)(x)=f(x)-g(x)$
    - $(fg)(x)=f(x)\cdot g(x)$
    - $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
  - Understand function composition: $(f\circ g)(x)=f(g(x))$

- 0.7 Trigonometric Functions:
  - Know the basics about the sine and cosine functions (graphs, values at 0, period, etc.)
  - Know the 4 other trig functions from pg 45 (tan, cot, sec, csc) in terms of sin and cos

## Chapter N: Introduction to Polynomial Calculus

*Because all of Chapter N is either review or covered in detail later in the course, it will not be set apart from the rest of the exam (problems will occur where we covered them in the book).

- N.1 Straight Lines:
  - Know how to find the slope of a (nonvertical) line through two points: $m=\frac{(y_2-y_1)}{(x_2-x_1)}$
  - Slope-Intercept form: y = mx + b    Point-Slope form: $y - y_1 = m(x - x_1)$
  - Use slope to determine whether lines are parallel or perpendicular

- N.2 Slope of a Curve:
  - BIG IDEA: slope of a curve depends on where you are on the curve!
  - Understand how to use and **memorize** the definition of the derivative:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

  - Remember! The value of the derivative at $x$ is the slope of the curve or graph $f(x)$ at $(x, f(x))$

- N.3 Derivative of a Polynomial:
  - Use the following rules to find the derivatives of polynomials:
    - For $f(x)=x^n$ , $f'(x)=nx^{n-1}$
    - $(cf(x))'=cf'(x)$
    - $(f(x)+g(x))'=f'(x)+g'(x)$

- N.4 Antiderivatives of Polynomials:
  - Use the following rules (the above in reverse) to find antiderivatives of polynomials
    - $F=\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$
    - $\int af(x)\,dx=a\int f(x)\,dx$
    - $\int (f(x)+g(x))dx=\int f(x)dx+\int g(x)\,dx$
  - **Understand position, velocity, and acceleration (see N.5 as well). You WILL see an application question of this type on the final (see 2.6 and 3.9).

- N.5 Definite Integrals:
  - Use the 2nd Fundamental Theorem of Calculus to find definite integrals $\int_a^b f(x)\,dx=F(b)-F(a)$ where $F$ is any antiderivative of $f$ ( $C=0$ for convenience)

## Chapter 1: Limits

- 1.1 Introduction to Limits:
  - Understand the intuitive meaning of a limit
  - Recall that a limit only exists if the right and left hand limits match (Thrm A pg 59)
  - Know how to evaluate basic limits limits that involve simply pugging in the value of x, or simplifying (ie: factoring and canceling zero denominators) so plugging in works

- 1.3 Limit Theorems:
  - Know and understand all of the limit rules in the Main Limit Theorem (Thrm A pg 68)
  - **Remember!** You are ALWAYS required to simplify complex limits to smaller limits that you know the limit of by applying this theorem.
  - Know that the Substitution Theorem from this section allows you to evaluate polynomials and rational functions with nonzero denominators by plugging in

- 1.4 Limits Involving Trigonometric Functions:
  - **<u>Memorize</u>**: $\lim_{x \to 0} \frac{\sin x}{x} = 1$ Big Idea: Simplify the trig functions into small pieces you know the limit of by using the limit rules from 1.3.
  - Key Tools:
    - Write any trig functions in terms of sin and cos
    - Multiply by 1 as the necessary $\frac{x}{x}$ to get pieces that look like $\frac{\sin x}{x}$

- 1.5 Limits at Infinity; Infinite Limits:
  - **<u>Memorize</u>**: $\lim_{x \to \infty} \frac{1}{x^k} = 0$ and $\lim_{x \to -\infty} \frac{1}{x^k} = 0$ (this is the key tool here)
  - Evaluate limits at infinity by rewriting functions with pieces like above (to do this, you divide <u>each term</u> by the largest $x^k$ in the denominator)

**<u>General Limit Information</u>**

- There are 3 cases for $x \to \pm\infty$ (*you must show the work to get the answer in every case!*):
  - degree numerator > degree denominator: limit goes to $\pm\infty$
  - degree numerator = degree denominator: limit is ratio of leading coefficients
  - degree numerator < degree denominator: limit is zero

(Note these 3 cases relate to horizontal asymptotes)

- There are 3 cases for $x \to c$ (*you must show the work to get the answer in every case!*):
  - The first step is always plug in $c$ (in your head if you want)
    - If you get a finite number back, you're done (limits in 1.1, 1.3, 1.4)
    - If you get $\frac{0}{0}$, you need to simplify more to find the limit. Simplify until you can plug in $c$ and get a finite number (1.1, 1.3, 1.4)
    - If you get $\frac{number}{0}$, the limit will go to $+\infty$ or $-\infty$ or it does not exist. To figure out which, you must evaluate the right and left hand limits. (Note that this case relates to vertical asymptotes)

- 1.6 Continuity of Functions:
  - Know and understand how to use the definition of continuity at a point

$$\lim_{x \to c} f(x) = f(c)$$

  - This definition requires 3 things (**<u>memorize</u>**):

1. $\lim_{x \to c} f(x)$ must exist 2. $f(c)$ must exist 3. $\lim_{x \to c} f(x) = f(c)$

**<u>Chapter 2: The Derivative</u>**

- 2.2 The Derivative:
  - Understand how to use and **<u>memorize</u>** the definition of the derivative:

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ There <u>WILL</u> be a question of this type on the exam!

  - Understand that differentiability implies continuity (but not vice versa)
  - Remember! Functions are not differentiable at corners and vertical tangents
- 2.3 Rules for Finding Derivatives:

- Know how to use the rules from this section to find derivatives (**<u>memorize</u>**)
  - Constant Rule $D_x(k)=0$
  - Identity Function Rule $D_x(x)=1$
  - Power Rule $D_x(x^n)=nx^{n-1}$
  - Constant Multiple Rule $D_x[k\cdot f(x)]=k\cdot D_x f(x)$
  - Sum/Difference Rule $D_x[f(x)\pm g(x)]=D_x f(x)\pm D_x g(x)$
  - Product Rule
    $D_x[f(x)g(x)]=f(x)D_x g(x)+g(x)D_x f(x)$ OR $(f\cdot g)'(x)=f(x)g'(x)+g(x)f'(x)$
  - Quotient Rule
    $D_x\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)D_x f(x)-f(x)D_x g(x)}{g^2(x)}$ OR $\left(\frac{f}{g}\right)'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{g^2(x)}$

- 2.4 Derivatives of Trigonometric Functions:
  - **<u>Memorize</u>**: $D_x(\sin(x))=\cos(x)$ and $D_x(\cos(x))=-\sin(x)$
  - Be able to use the above with the rules from 2.3 to find derivatives of trig functions

- 2.5 The Chain Rule:
  - Understand when and how to apply the chain rule (**<u>memorize</u>**):

    $$(f\circ g)'(x)=f'(g(x))g'(x)$$

  - Remember! You must use the chain rule any time that function composition is involved

- 2.6 Higher-Order Derivatives:
  - Be able to take higher order derivatives, including those involved in applied questions involving position, velocity, and acceleration (see N.4, 3.9).
  - Know how position, velocity and acceleration are related. This WILL be on the final!

- 2.7 Implicit Differentiation:
  - Steps: 1. Take the derivative of both sides (using dy/dx notation!) of the equation. 2. Move all terms involving dy/dx to one side and solve for dy/dx
  - Remember! y is a function of x (y(x)), so you must use the chain rule to differentiate any terms involving y.

- 2.8 Related Rates:
  - There WILL be 1 related rates question on the exam. It will be similar to questions given in the homework, on the take home quiz, and exam 2. Complex formulas will be given if needed (but you need to know similar triangles and the Pythagorean theorem).
  - Steps:
    - Draw a picture that is valid <u>for all time>0</u>. <u>Label</u> the values you know, and chose variables for and <u>label</u> the values you don't know, but need.
    - State what is given and what you want to find. This information will be in the form of derivatives with respect to t.
    - Relate the variables by writing an equation that is true for ALL time>0. Do NOT plug in what you know!!
    - Implicitly differentiate with respect to time.
    - Finally, plug in what you know and solve for what you want to find! Note: you may need to eliminate an extra variable by using the Pythagorean theorem or similar triangles.

## Chapter 3: Applications of the Derivative

- 3.1 Maxima and Minima:
  - Understand the Critical Point Thrm: If f(c) is an extrema, then c must be a critical point. Either c is
    - an endpoint of I
    - a stationary point of f ( $f'(c)=0$ )
    - a singular point of f ( $f'(c)$ does not exist)
  - Note that this does NOT mean that all critical points are extrema

- 3.2 Monotonicity and Concavity (**memorize)**:
  - Understand the Monotonicity Thrm
    - $f'(x)>0$ then $f$ is increasing
    - $f'(x)<0$ then $f$ is decreasing
  - Understand the Concavity Thrm
    - $f''(x)>0$ then $f$ is concave up
    - $f''(x)<0$ then $f$ is concave down
  - Know what inflection points are and how to find them

- 3.3 Local Extrema:
  - Know the difference between local and global extrema
  - Know and understand when and how to apply the first derivative test (**memorize**)
    - $f'(x)>0$ on the left of c and $f'(x)<0$ on the right of c, means c is a local max
    - $f'(x)<0$ on the left of c and $f'(x)>0$ on the right of c, means c is a local min
    - If $f'(x)$ has the sign same on both sides of c, c is not a local extreme value
  - Know and understand when and how to apply the second derivative test
    - If $f''(c)<0$ then $f(c)$ is a local max
    - If $f''(c)>0$ then $f(c)$ is a local min
    - Note that if $f''(c)=0$ the test is inconclusive

- 3.5 Graphing Functions Using Calculus:
  - **Steps**: Precalculus:
    - Find the domain of the function
    - Test for symmetry (is the function even or odd)
    - Find the intercepts (both x and y)

    Calculus:
    - Find the critical points
    - Find out where the function is increasing and decreasing
    - Find local minima and maxima
    - Find out where the graph is concave up and concave down
    - Find any inflection points
    - Find the asymptotes

*There WILL be a question about the material from sections 3.1, 3.2, 3.3, and 3.5 on the final*

- 3.8 Antiderivatives:
  - F is an antiderivative of f if $F'(x)=f(x)$
  - Rules (**memorize**):
    - Power Rule: $F=\int x^r dx=\frac{x^{r+1}}{r+1}+C$ (r is any rational number except -1)
    - $\int kf(x)dx=k\int f(x)dx$

- $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int \sin(x)\,dx = -\cos(x) + C$ and $\int \cos(x)\,dx = \sin(x) + C$
- General Power Rule: $F = \int [g(x)]^r g'(x)\,dx = \frac{[g(x)]^{r+1}}{r+1} + C$ (undoing chain rule)

- Remember! Don't forget the $+C$ on the antiderivatives!!

- 3.9 Introduction to Differential Equations:
  - Be able to use Separation of Variables to solve first order, separable DEs
    - Steps: separate the 2 variables, integrate both sides (don't forget the +C!), use algebra to solve for the dependent variable (ie: y for y(x))
  - There WILL be a question on the final relating position, velocity, and acceleration. See HW #17 for an example of how this relationship is applied for integration.

**Chapter 4: The Definite Integral**

- 4.1 Introduction to Area:
  - Understand the basics of sigma notation, be able to use it to work with sums
  - If you have to simplify a sum, you are required to show the work required for using linearity. Do NOT memorize the special sum rules, but know how to use them!
    - $\sum_{i=1}^{n} c = nc$
    - $\sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i$
    - $\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

- 4.2 The Definite Integral:
  - Understand integration in terms of the Riemann sum: Sum up the areas of n rectangles under a curve, and take the limit as the number of rectangles goes to infinity. Thus, the definite integral is the area under the curve.
  - You will be given appropriate formulas if asked to solve a question from this section.
  - Remember that area is signed area here (area below the curve is negative, but area is always positive, so we change the sign. This does NOT apply for area between curves!)
    - $\int_a^b f(x)\,dx = A_{up} - A_{down}$
  - Know how to use the following properties (**memorize**):
    - $\int_a^b f(x)\,dx = -\int_a^b f(x)\,dx$
    - $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

- 4.3 The 1st Fundamental Thrm of Calculus:
  - It has the word fundamental in the title: it **WILL** be on the final!
  - Understand the 1st Fundamental Thrm of Calculus (**memorize**)
    - $\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$
    - The location of the x matters. If x is a function, the chain rule is required!
    - **Remember!** The best part about this theorem is that there is NO actual integration involved in finding the answer!
  - Understand linearity of integration
    - $\int_a^b kf(x)\,dx = k \int_a^b f(x)\,dx$
    - $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

- 4.4 The 2nd Fundamental Thrm of Calculus and the Method of Substitution:
  - It has the word fundamental in the title: both the 2nd Fund. Thrm and substitution for definite and indefinite integrals **WILL** be on the final!
  - Recall the 2nd Fund. Thrm from Chapter N (**memorize**): $\int_a^b f(x)\,dx = F(b) - F(a)$
  - Use substitution to evaluate indefinite integrals involving function composition
    - $\int f(g(x))g'(x)\,dx = \int f(u)\,du = F(u) + C = F(g(x)) + C$ let $u = g(x)$ so $du = g'(x)\,dx$
    - **Remember**! Define u=g(x) based on the function composition (it is the inside function), and then FIND du from this u (don't just read it off from the integral...it may not match exactly!). You may need to multiply or divide by a constant to get the du term to fit your integral.
  - Use substitution to evaluate definite integrals involving function composition
    - $\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du = [F(u)]_{g(a)}^{g(b)} = F(g(b)) - F(g(a))$
    - **Remember!** Use the same methods for substitution as for indefinite integrals, but change your limits of integration to be in terms of u=g(x).

## Chapter 5: Applications of the Integral

- 5.1 The Area of a Plane Region:
  - Be able to find the area of a plane region
    - Steps:
      - sketch the region
      - slice into thin pieces, label and typical piece (choose dx or dy)
      - approximate the area of the typical piece as if it were a rectangle
        - Remember! If it's the area between 2 curves, use f(x)-g(x) as the length of the rectangle where f(x)> g(x)
        - Remember! The sign of the answer automatically works out if you subtract correctly (you don't correct for being below the x-axis)!
      - find the limits of integration (you need the intersection points of the 2 graphs)
      - integrate: Area = $\int_a^b$ length * width (width is dx or dy) (**memorize**)

- 5.2 Volumes of Solids: Disks, Washers:
  - Be able to use the disk and washer methods to find the volume of solids
  - Disk Method
      - sketch graph
      - decide on dx or dy thickness
      - find limits of integration
      - determine the area for a typical slice (slices are always circular, $A = \pi r^2$ )
      - integrate: Volume = $\int_a^b$ Area(x) dx (or A(y) and dy) (**memorize**)
  - Washer Method
      - same as above, but $A = \pi[r_{outer}^2 - r_{inner}^2]$ since the slice is a washer (**memorize**)
      - **Remember**! Measure the radius from the axis of revolution!

  - **Remember!** The slice looks perpendicular to the axis of revolution for these methods (ie: revolve about x-axis, use dx slices, revolve about y axis, use dy slices)
  - **Remember**! The integration limits are in terms of the variable of thickness (x for dx or y for dy) and only range in the original area (not the revolved area).

- 5.3 Volumes of Solids: Shells:
  - Be able to use the shell method to find the volume of solid
  - **Remember**! Shell and washer methods are best for similar problems (those with open areas in the middle of the revolved shape).
  - Shell Method
    - sketch graph
    - decide on dx or dy thickness
    - find limits of integration
    - determine the radius and height for a typical shell (both depend on your variable...there are infinitely many shells inside the volume)
    - Integrate: Volume = $2\pi\int_a^b$ radius*height dx (or dy) (**memorize**)
  - **Remember!** The shell looks parallel to the axis of revolution (ie: revolve about x-axis, use dy slices (shell on side), revolve about y axis, use dx slices (shell upright))
  - **Remember**! The integration limits are in terms of the variable of thickness (in x for dx or y for dy) and only range in the original area (not the revolved area).

- 5.4 Length of a Plane Curve:
  - Be able to use the 3 arc length equations to find the arc length of various curves (do NOT memorize, but know when to use each formula).
    - L= $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ for $a \le t \le b$ (parametric equations, x=f(t), y=g(t) )
    - L= $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ for $a \le x \le b$ (not parametric, y=f(x) )
    - L= $\int_c^d \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy$ for $c \le y \le d$ (not parametric, x=g(y) )

- 5.5 Work and Fluid Force:
  - Be able to solve basic force questions involving springs (see HW for what to focus on).
  - Remember! Work = Force * Distance
  - Take variable force into account by summing over small distances
    - W= $\int_a^b F(x)\,dx$
  - Hooke's Law: F(x)=kx, where k is the spring constant

*There WILL be one question from 5.4 OR 5.5
(worth a smaller amount of points than the other questions)*