Even numbered problems

#20/673 $F(x,y) = x^3 - xy^2 - y^4$ $\begin{cases} x = 2\cos 3t \\ y = 3\sin t \end{cases}$ $\left.\frac{dF}{dt}\right|_{t=0} = ?$

Chain rule: $\frac{dF}{dt} = \frac{\partial F}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt}$.

$\frac{\partial F}{\partial x} = 3x^2 - y^2$; $\frac{\partial F}{\partial y} = -2xy - 4y^3$; $\frac{dx}{dt} = -6\sin 3t$, $\frac{dy}{dt} = 3\cos t$

At $t=0$: $\begin{cases} x = 2 \\ y = 0 \end{cases}$ so $\left.\frac{\partial F}{\partial x}\right|_{t=0} = 12$, $\left.\frac{\partial F}{\partial y}\right|_{t=0} = 0$, and $\left.\frac{dx}{dt}\right|_{t=0} = 0$

$\left.\frac{dy}{dt}\right|_{t=0} = 3$

Then $\left.\frac{dF}{dt}\right|_{t=0} = 12\cdot 0 + 0\cdot 3 = \boxed{0}$.

#24/673 $r = 10$, $dr = \pm 0.02$; $h = 6$, $dh = \pm 0.01$

$V = \pi r^2 h$ $\quad V(10,6) = \pi\cdot 10^2\cdot 6 = \boxed{600\pi}$

The error: $dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh = (2\pi rh)dr + (\pi r^2)dh$

error in V $\approx dV = (2\pi\cdot 10\cdot 6)(\pm .02) + (\pi 10^2)(\pm 0.01)$

$= (\pm 2.4\pi) + (\pm \pi)$

$= \boxed{\pm 3.4\pi}$

#22/634 $f(x,y) = \begin{cases} \frac{\sin(xy)}{xy}, & \text{if } xy \neq 0 \\ 1, & \text{if } xy = 0. \end{cases}$

Whenever $xy \neq 0$, $f(x,y)$ is a combination of continuous elementary functions $(\sin z, \frac{1}{z})$, so it is continuous. The question is if it is also continuous for (x,y) with $xy = 0$.

Let (x_0, y_0) be a point such that $x_0 y_0 = 0$.

$\lim_{(x,y)\to(x_0,y_0)} \frac{\sin(xy)}{xy} = \lim_{t\to 0} \frac{\sin t}{t} = 1 = f(x_0,y_0)$.

$t = xy$

Since the limit exists, and it equals the value of the function, f must be continuous at (x_0,y_0). In conclusion, f is $\boxed{\text{continuous everywhere}}$