Math 2210-1. Practice Final. Fall 2010.

Name: ____________________ December 7, 2010

Problem 1: _____ /35

Problem 2: _____ /33

Problem 3: _____ /32

Problem 4: _____ /30

Problem 5: _____ /20

Total: _____ /150

Instructions: The exam is closed book, closed notes and calculators are not allowed. You are only allowed one letter-size sheet of paper with anything on it.

You will have two hours for this test. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

Problem 1.

(1) Find the equation of the tangent plane to the surface $x^2+y^2-z^2=4$ at the point $(2,1,1)$.

(2) Find the parametric equations for the tangent line to the curve $\vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}$, at the point $P(2) = (2, 2, \frac{8}{3})$.

(3) Is the line in (2) parallel to the plane in (1)?

Problem 2. Consider the function $f(x,y) = x^2 + y^2 - xy$ on the closed disk $S : x^2 + y^2 \leq 9$.

(1) Find the critical points in the interior of S and decide if they give local maxima, minima, or neither.

(2) Find the maximum and minimum values of $f(x,y)$ on the boundary of S.

(3) Using the results in (1) and (2), conclude what the global maximum and minimum values of $f(x,y)$ on S are.

Problem 3.

(1) Draw a clear picture of the wedge cut from the cylinder $y^2 + z^2 = 4$ in the first octant by the planes $y = \frac{x}{2}$ and $x = 4$.
(2) Find the volume of the cylindrical wedge described in (1). Evaluate the integral!
(3) Set up completely an integral which calculates the cylindrical surface area of the wedge in (1). Do not evaluate the integral! (*Extra credit*: Evaluate the integral.)

Problem 4. Let C be a curve between the points $(0,0)$ and $(1,2)$ and consider the line integral

$$\int_C y^2 dx + 2xy dy.$$

(1) Show that the line integral is independent of the path, i.e., independent of the particular form of C.
(2) Evaluate the integral by choosing a particular (simple) C.
(3) Evaluate the integral by using the fundamental theorem of calculus for line integrals.

Problem 5. Let $\vec{F}$ be the vector field

$$\vec{F} = xy\vec{i} + (x^2 + y^2)\vec{j}.$$

(1) Compute $curl\vec{F}$ and $div\vec{F}$.
(2) Calculate the flux of $\vec{F}$ across the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$.