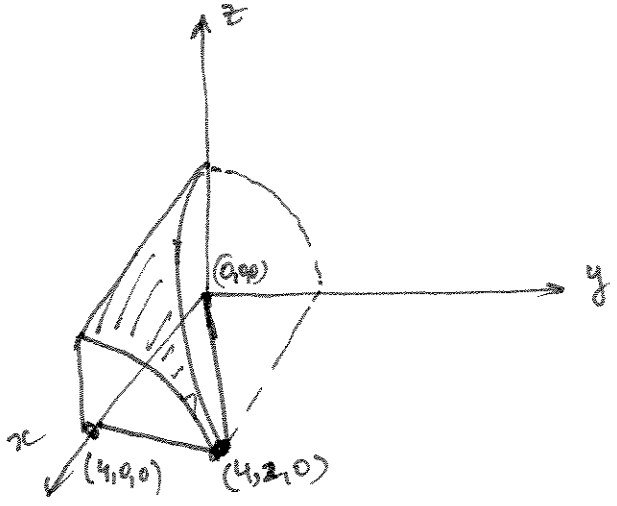
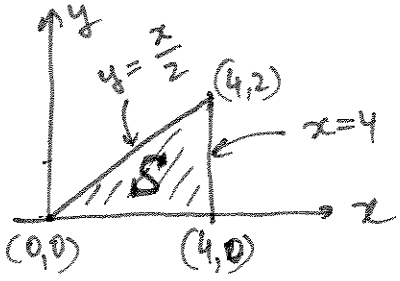
Problem 3.

(1) Draw a clear picture of the wedge cut from the cylinder $y^2 + z^2 = 4$ in the first octant by the planes $y = \frac{x}{2}$ and $x = 4$.
(2) Find the volume of the cylindrical wedge described in (1). Evaluate the integral!
(3) Set up completely an integral which calculates the cylindrical surface area of the wedge in (1). Do not evaluate the integral! (*Extra credit*: Evaluate the integral.)

(1)



(2) The projection onto the xy-plane is:



The volume is $V = \iint_S z\,dA = \iint_S \sqrt{4-y^2}\,dA = \int_0^2 \int_{2y}^4 \sqrt{4-y^2}\,dx\,dy$

$$V = \int_0^2 (4-2y)\sqrt{4-y^2}\,dy = 2\int_0^{\frac{\pi}{2}} (4-4\sin\theta)\cos^2\theta\,d\theta$$

$y = 2\sin\theta$
$dy = 2\cos\theta\,d\theta$

$= \dots =$ old solutions $\boxed{2\pi - \frac{8}{3}}$

(3) cylindrical Area $= \iint_S \frac{2}{\sqrt{4-y^2}}\,dA = \boxed{\int_0^2 \int_{2y}^4 \frac{2}{\sqrt{4-y^2}}\,dx\,dy}$