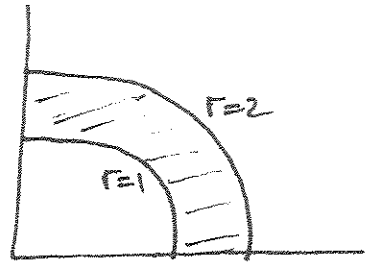
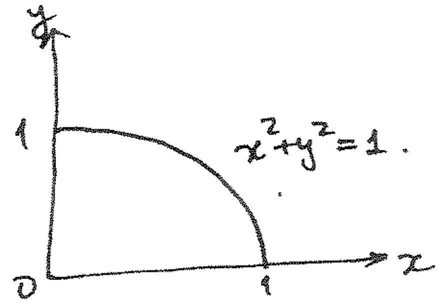
22/p.695 $\iint_S y \, dA$



In polar coordinates, the two circles are $r=1$ and $r=2$

$$\int_0^{\frac{\pi}{2}} \int_1^2 r^2 \sin\theta \, dr \, d\theta = \frac{7}{3} \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta = \boxed{\frac{7}{3}}.$$

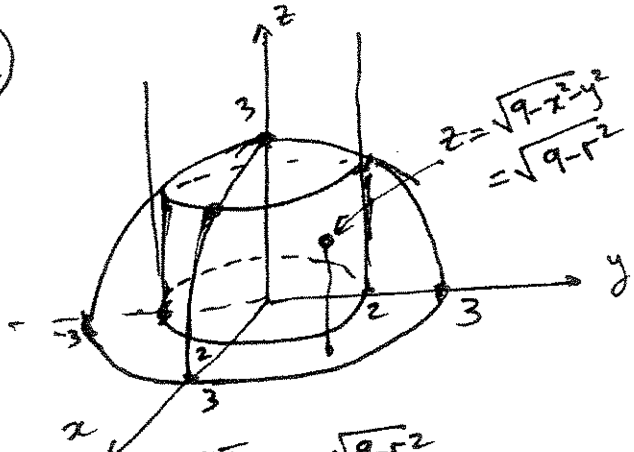
24/p.695 $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2+y^2) \, dx \, dy.$



$$= \int_0^{\frac{\pi}{2}} \int_0^1 \sin r^2 \, r \, dr \, d\theta$$

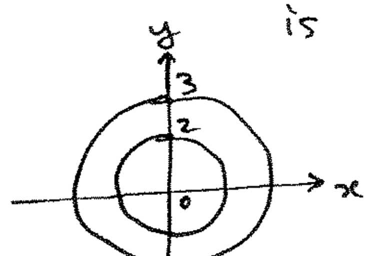
$$= \frac{\pi}{2} \left[-\frac{1}{2} \cos(r^2) \right]_0^1 = \cancel{\boxed{\frac{\pi}{4} \sin(1)}} = \boxed{\frac{\pi}{4}(1-\cos 1)}$$

8/p.717



The volume is <u>outside</u> the cylinder $x^2+y^2=4$.

The projection onto the x-y-plane is



$$\text{Volume} = \iiint_S dV = \int_0^{2\pi} \int_2^3 \int_0^{\sqrt{9-r^2}} dz \, r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_2^3 \sqrt{9-r^2} \, r \, dr \, d\theta =$$

$u = 9-r^2$

$du = -2 r \, dr$

$r=2 \to u=5$

$r=3 \to u=0$.

$$= \int_0^{2\pi} \int_5^0 -\frac{1}{2} u^{1/2} \, du \, d\theta = \frac{1}{2} \int_0^{2\pi} \left. \frac{2}{3} u^{3/2} \right]_0^5 d\theta = \frac{1}{3} 5^{3/2} \cdot 2\pi$$

20/p.717

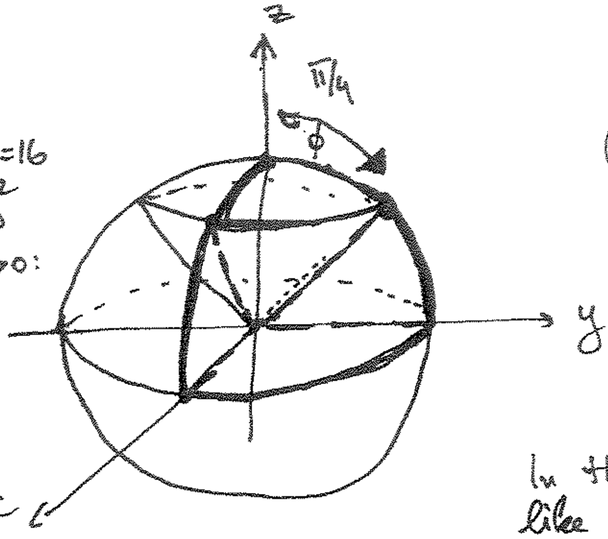
sphere $x^2+y^2+z^2=16$

cone $z^2=x^2+y^2$

intersection for $z>0$:

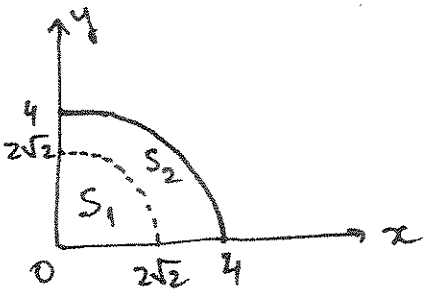
$z^2=8 \quad z=2\sqrt{2}$

$x^2+y^2=8$



The volume is four times the piece in the first octant. (One solution would be to compute the volume inside the cone and subtract it from the volume of the sphere. We will set up the integral directly instead.)

In the xy-plane the projection looks like



As a double integral:

$$4\iint_S z\,dA = 4\iint_{S_1} \sqrt{x^2+y^2}\,dA \;+\; 4\iint_{S_2}\sqrt{16-x^2-y^2}\,dA \ldots \text{ etc.}$$

As a triple integral:

$$V=4\iiint_V dV \underset{\text{spherical}}{=} 4\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^4 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

the easiest solution

$$= 2\pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^4 \rho^2\sin\phi\, d\rho\, d\phi = 2\pi \int_{\frac{\pi}{4}}^{\pi/2} \left.\frac{\rho^3}{3}\right]_0^4 \sin\phi\, d\phi$$

$$= \frac{128\pi}{3}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\phi\, d\phi = \frac{128\pi}{3}\Big[-\cos\phi\Big]_{\frac{\pi}{4}}^{\pi/2}$$

$$\text{Vol} = \frac{128\pi\sqrt{2}}{6} = \boxed{\frac{64\pi\sqrt{2}}{3}}$$

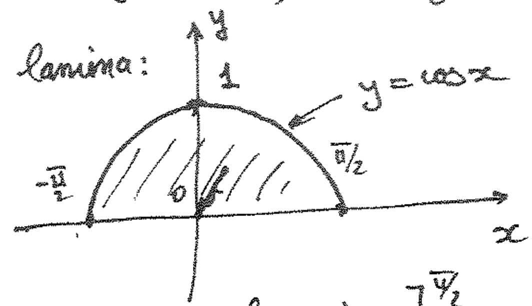
(18/699)

$$\int_{-\pi/2}^{\pi/2} \int_0^{\cos x} k \, dy \, dx$$

$x \in [-\pi/2, \pi/2]$
for every x, $y \in [0, \cos x]$

The density is $\delta(x,y) = k$
(constant)

lamina:



$$\text{mass} = k \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} dy \, dx = k \int_{-\pi/2}^{\pi/2} \cos x \, dx = k \sin x \Big]_{-\pi/2}^{\pi/2} = \boxed{2k}$$

$$M_y = k \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} x \, dy \, dx = k \int_{-\pi/2}^{\pi/2} x \cos x \, dx = 0$$ (expected from symmetry)

$x \cos x$ odd function on $[-\pi/2, \pi/2]$.

$$\boxed{\bar{x} = 0}$$

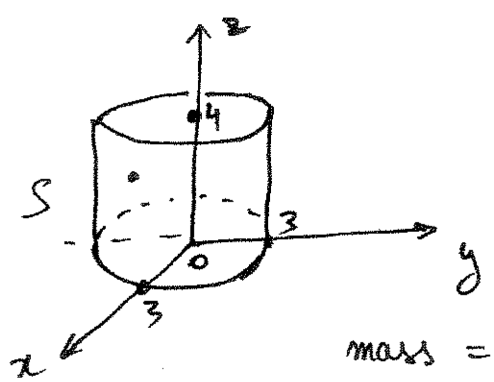
$$M_x = k \int_{-\pi/2}^{\pi/2} \int_0^{\cos x} y \, dy \, dx = \frac{k}{2} \int_{-\pi/2}^{\pi/2} \cos^2 x \, dx$$

$$= \frac{k}{4} \int_{-\pi/2}^{\pi/2} [1 + \cos 2x] \, dx = \frac{k\pi}{4} + \cancel{\frac{k}{4} \left[\frac{1}{2} \sin 2x\right]_{-\pi/2}^{\pi/2}}^{\,0}$$

$$\Rightarrow \boxed{\bar{y} = \frac{\pi}{8}}$$

Center of mass $\boxed{\left(0, \frac{\pi}{8}\right)}$.

(26/712)



$\delta(x,y,z) = k(x^2+y^2+z^2)$

Because of the symmetry of the object and of the density, we see that $\bar{x} = 0$, $\bar{y} = 0$.

$$\text{mass} = \iiint_S \delta(x,y,z) \, dV$$

$$m \underset{\substack{\uparrow \\ \text{cylindrical} \\ \text{coords}}}{=} k \int_0^{2\pi}\int_0^3\int_0^4 (r^2+z^2)\, r \, dz\, dr\, d\theta = 2\pi k \int_0^3 \int_0^4 (r^3 + z^2 r)\, dz\, dr$$

$$= 2\pi k \int_0^3 \left[r^3 z + r\frac{z^3}{3} \right]_{z=0}^{z=4} dr$$

$$= 2\pi k \int_0^3 \left[4r^3 + \frac{64}{3} r \right] dr = 2\pi k \left[r^4 + \frac{32}{3} r^2 \right]_0^3$$

$$= 2\pi k \left(81 + 32 \cdot 3 \right) = \boxed{354 \pi k}$$

$$M_{xy} = k \int_0^{2\pi}\int_0^3\int_0^4 z(r^2+z^2)\, r\, dz\, dr\, d\theta$$

$$= 2\pi k \int_0^3 \int_0^4 \left[r^3 z + r z^3 \right] dz\, dr$$

$$= 2\pi k \int_0^3 \left[r^3 \frac{z^2}{2} + r \frac{z^4}{4} \right]_{z=0}^{z=4} dr$$

$$= 2\pi k \int_0^3 (8r^3 + 64 r)\, dr = 2\pi k \left[2r^4 + 32 r^2 \right]_{r=0}^{r=3}$$

$$= 2\pi k \left[162 + 96 \right) = \boxed{516 \pi k}$$

So $\bar{z} = \dfrac{516 \cancel{\pi k}}{354 \cancel{\pi k}} \Rightarrow \bar{z} = \dfrac{86}{59}$