

## Discrete Math 2200. Problem Set 7

**Due date:** Tuesday, November 4, in class. Late homeworks are not accepted, except for a medical or some other university approved reason.

Unless specified otherwise, the numbering of the exercises below is as in the textbook (Rosen, ed. 6).

**Problem 1.** Prove by mathematical induction the formulae:

(a)

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6},$$

for all positive integers  $n$ .

(b)

$$1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3},$$

for all positive integers  $n$ .

**Problem 2.** Prove by induction the following inequalities:

(a)  $n! < n^n$ , for all integers  $n \geq 2$ .

(b)

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

for all integers  $n \geq 2$ .

**Problem 3.** Solve by induction the following division problems:

(a) 3 divides  $n^3 + 2n$  for every positive integer  $n$ .

(b) 133 divides  $11^{n+1} + 12^{2n-1}$ , for every positive integer  $n$ .

**Problem 4.** (Sets)

(a) ex. 40 page 281.

(b) ex. 45 page 281 (will be graded).

**Problem 5.** (strong induction) ex. 12 page 292.