Discrete Math 2200. Problem Set 7

Due date: Tuesday, November 4, in class. Late homeworks are not accepted, except for a medical or some other university approved reason.

Unless specified otherwise, the numbering of the exercises below is as in the textbook (Rosen, ed. 6).

Problem 1. Prove by mathematical induction the formulae:
(a) \[ 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}, \]
for all positive integers \( n \).
(b) \[ 1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}, \]
for all positive integers \( n \).

Problem 2. Prove by induction the following inequalities:
(a) \( n! < n^n \), for all integers \( n \geq 2 \).
(b) \[ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}, \]
for all integers \( n \geq 2 \).

Problem 3. Solve by induction the following division problems:
(a) 3 divides \( n^3 + 2n \) for every positive integer \( n \).
(b) 133 divides \( 11^{n+1} + 12^{2n-1} \), for every positive integer \( n \).

Problem 4. (Sets)
(a) ex. 40 page 281.
(b) ex. 45 page 281 (will be graded).

Problem 5. (strong induction) ex. 12 page 292.