Problem 1. Prove that if $p$ is a prime number, then the only solutions of $x^2 \equiv 1 \pmod{p}$ are integers $x$ such that $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$.

Problem 2. Apply the Chinese Remainder algorithm to solve the following systems of congruences (only the system in a) will be graded):

- a) $\begin{cases} x \equiv 2 \pmod{3} \\ x \equiv 1 \pmod{4} \\ x \equiv 3 \pmod{5} \end{cases}

- b) $\begin{cases} x \equiv 5 \pmod{6} \\ x \equiv 1 \pmod{5} \\ x \equiv 2 \pmod{7} \end{cases}


Problem 4.

- a) Prove that $(a + 1)^n \equiv 1 \pmod{a}$, for any positive integers $a, n$, $a \geq 2$.
- b) ex 27/245 (Although it is an odd numbered exercise, you must turn it in, and it will be graded. At some point, you may need to use part a)).
- c) ex 28/245.