Math 2200. Discrete Mathematics.
Review session. 09.23.08

A. Propositional logic

(1) determine if a proposition is a tautology, contradiction, or contingency, using truth tables or logical laws;
(2) translate from English sentences to propositional logic using operations and quantifiers; negations;
(3) determine if an argument is valid or not using rules of inference; deduce conclusions from premises.

B. Sets

(1) prove that two sets are equal, or one is a subset of another;
(2) identify the elements of a set, compute power sets and cartesian products;
(3) set operations and identities, union, intersection, complement;
(4) cardinality, easy examples of countable/uncountable.

C. Functions

(1) domain, codomain, image (range);
(2) check if a function is one-to-one, onto, bijective, increasing, decreasing, or find examples of such functions;
(3) find the inverse function of a given bijective function;
(4) composition of functions;
(5) simple identities involving the floor function.

D. (Simple) Examples of proof techniques

(1) direct proofs;
(2) proof by contrapositive;
(3) proof by contradiction;
(4) proof by cases;
(5) counterexamples, existence proofs.

Examples

The following are some supplemental examples (to homework, class examples), and by no means the only examples required for the test.

A. Propositional logic

(1) Prove that $(\overline{q} \wedge (p \to q)) \to \overline{p}$ is a tautology.
(2) Use quantifiers: "There is a building on the campus of some college in the United States in which every room is painted white". What is the negation?
(3) What are the logical conclusions of the following premises: "All foods that are healthy to eat do not taste good." "Tofu is healthy to eat." "You only eat what tastes good." "You don't eat tofu." "Cheeseburgers are not healthy to eat."
(4) Is the argument valid?: "No man is an island." "Manhattan is an island. Therefore Manhattan is not a man."

B. Sets

(1) List the elements of the set $\{x | x \text{ is a real number such that } x^2 = 1 \text{ or } x^2 = 5\}$.
(2) Show that $(B - A) \cup (C - A) = (B \cup C) - A$.
(3) What can you say about A, B if: a) $A - B = A$; b) $A - B = B - A$.
(4) Let $A_i = \{\dots, -2, -1, 0, 1, \dots, i\}$, find $\cup_{i=1}^{\infty} A_i$, $\cap_{i=1}^{\infty} A_i$.
(5) Show that the set of natural numbers divisble by 5 is countable. Show that the set of prime numbers is countable. Show that the set of irrational numbers is uncountable.

C. Functions

(1) Is $f : \mathbb{R} \to \mathbb{R}$ one-to-one, onto?: $f(x) = 2x + 1$; $f(x) = x^3$; $f(x) = x^2 + 1$; $f(x) = \frac{x^2+1}{x^2+2}$. What is the inverse if f is bijective?
(2) Prove that for every positive real number x, $\lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$.

D. Proof techniques

(1) If n is odd, then n^2 is odd.
(2) If n^2 is divisible by 3, then n is divisible by 3.
(3) $\sqrt{2}$ is irrational. At least 8 of any 50 days chosen fall in the same day of the week.
(4) $\max(x, y) + \min(x, y) = x + y$. $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$.
(5) $n^2 \geq 2n$ for all natural numbers n. Between any two distinct rational numbers, there exists a rational number.