

**Math 2200. Discrete Mathematics. Lecture 8.**  
**09.18.08**

1. Prove that if  $m$  is an integer, then  $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$  if  $x$  is not an integer, and  $\lfloor x \rfloor + \lfloor m - x \rfloor = m$ , if  $x$  is an integer.

2. Let  $A$ ,  $B$ , and  $C$  be sets. Show that  $(A - B) - C = (A - C) - (B - C)$ .

3. Determine whether each of these functions from  $\mathbb{Z}$  to  $\mathbb{Z}$  is one-to-one, or onto.

a)  $f(n) = n - 1$ ;

b)  $f(n) = n^2 + 1$ ;

c)  $f(n) = n^3$ ;

d)  $f(n) = \lfloor \frac{n}{2} \rfloor$ .

4. Find  $f \circ g$  and  $g \circ f$ , where  $f(x) = x^2 + 1$  and  $g(x) = x + 2$  are functions from  $\mathbb{R}$  to  $\mathbb{R}$ .

5. Let  $f$  be a function from the set  $A$  to the set  $B$ . Let  $S$  and  $T$  be subsets of  $A$ . Show that

a)  $f(S \cup T) = f(S) \cup f(T)$ ;

b)  $f(S \cap T) \subseteq f(S) \cap f(T)$ .

6. Suppose  $A$  and  $B$  are two sets such that their power sets are equal,  $P(A) = P(B)$ . Does it follow that  $A = B$ ?