

Math 2200-1. Final Exam. Fall 2008.

Name: _____

December 17, 2008

Problem 1: _____/20

Problem 2: _____/20

Problem 3: _____/20

Problem 4: _____/20

Problem 5: _____/20

Problem 6: _____/20

Total: _____/120

Instructions: The exam is closed book, closed notes and calculators are not allowed. You are only allowed three letter-size sheets of paper with anything on them.

You will have 120 minutes for this test. The point value of each problem is written next to the problem - use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

Problem 1 (Propositional Logic). Prove that the propositions

- (i) $(p \rightarrow q) \wedge (p \rightarrow r)$ and $(\bar{q} \vee \bar{r}) \rightarrow \bar{p}$ are logically equivalent.
- (ii) $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

Problem 2 (Induction). Prove by mathematical induction that

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} < \frac{3}{2} - \frac{1}{n^2}, \quad \text{for all } n \geq 2.$$

Problem 3 (Sets).

(i) Prove that $(A_1 \cup A_2) - B = (A_1 - B) \cup (A_2 - B)$, for every sets A_1, A_2, B .

(ii) Prove by mathematical induction that

$$(A_1 \cup A_2 \cup \cdots \cup A_n) - B = (A_1 - B) \cup (A_2 - B) \cup \cdots \cup (A_n - B),$$

for every sets A_1, A_2, \dots, A_n , $n \geq 2$, and B .

Problem 4 (Basic Proofs).

- (i) Prove that x is irrational if and only if $3x + 2$ is irrational.
- (ii) Assume that n is a perfect square. Prove that $n + 6$ is not a perfect square.

Problem 5 (Modular Arithmetic).

- (i) What does it mean for a residue c to be an inverse of a modulo m ? What is the condition that a has an inverse modulo m ?
- (ii) Does the system of congruences

$$\begin{cases} x \equiv 1 \pmod{2} \\ x \equiv 2 \pmod{5} \\ x \equiv 3 \pmod{7} \end{cases}$$

have solutions? If it does, find explicitly the smallest positive solution.

Problem 6 (Counting).

- (i) How many positive integers not exceeding 200 that are divisible by 3 or 5 are there?
- (ii) What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

Extra credit. (Attempt to solve it only if you finished the rest of the test.)

How many surjective functions from a set with 10 elements to a set with 6 elements are there?
(Hint: count how many *non*-surjective functions there are.)

⁰Thank you for your hard work this semester. Happy Holidays!